
Voters' dissatisfaction, abstention and entropy: analysis in European countries

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Abstract. This paper intends to explore the utilization of entropy through politics and election results, an area just slightly explored. It generalizes the interpretation of entropy, considering it a measure of dissatisfaction and disillusion of populations in relation to politics. Some phenomena like the increase of abstention in a country, consequence of the dissatisfaction of population and of their alienation in relation to politics could be detected and analysed. This discontentment could result, for example, in the appearance of new political parties, with more division of votes and increasing entropy (result of the discontentment and uncertainty by electors). Absolute majorities, while imply less dispersion of votes, are synonym of more confidence in a given party, making a reduction of entropy.

Keywords: Entropy, electoral results, satisfaction and dissatisfaction of population

JEL Classifications: C19 – Other; D72 – Economic Models of Political Processes: Rent-Seeking, Elections, Legislatures, and Voting Behavior

1. Introduction

Entropy is a measure of information and uncertainty which has been used recently in different areas, besides of its original utilization in physics, creating a new research area: Econophysics - the application of physical methods with the objective of solving economic problems. This research area started with the study of financial markets but was extended to other themes, like the insurance market, microeconomics, utility functions or macroeconomics. For example, Candeal *et al* (2001) show that exists a great similarity between theories of entropy and utility. There are also several examples that show the analogies between physics and economics (see, for example, Ausloos *et al* 1999, Stanley *et al* 2001, or Bouchaud 2002, among others). Studying the social problem, Bailey (1990, 1993) introduces the concept of Social Entropy Theory, applying it to Sociology. Furthermore, Chen (2003) uses entropy in a psychology context.

The concept of entropy had also been used in election studies. Kirchgässner and Schimmelpfennig (1992) use it in an indirect form to study electoral participation. According to some authors, how closer is an election, more participated is that election, because voters think that have more decision power. In that paper, entropy is used as a measure of closeness of results and it should be positively correlated with electoral participation. Gill (2005) uses entropy as a measure of uncertainty with the objective of trying to explain voting decision of

electors that effectively vote. Also Coleman (2004) uses entropy in the electoral context, in this case to measure what he calls for conformity: "many people vote to conform with the social norm that voting is a civic duty." The author concludes that there is a conformist vote behaviour by electors. So, abstention is voter nonconformity. Our analysis is similar, and we add that the increasing abstention could also be seen as dissatisfaction of electors. Considering the "Contract with America", a document of the Republican Party in 1994 USA elections to clarify American's opinions, Gill (2005) analyses if such phenomena had changed uncertainty levels of electors, concluding that uncertainty is important in their decisions. Furthermore, Antweiler (2006) uses maximum entropy to analyse migration votes in two consecutive elections, to capture which electors changed their vote in the first to the second election¹.

This paper intends to explore the utilization of entropy through politics and election results, but with different objectives. It generalizes the interpretation of entropy, considering it a measure of dissatisfaction and disillusion of populations in relation to politics, using data for legislative elections².

Some phenomena like the increase of abstention in a country, consequence of the dissatisfaction of population and alienation in relation to politics could be detected and analysed. This discontentment could result, for example, in the appearance of new political parties, with more division of votes and increasing entropy (result of the discontentment and uncertainty by electors³). Absolute majorities, while imply less dispersion of votes, are synonym of more confidence in a given party, making a reduction of entropy.

Section 2 explains the concept of entropy and how this measure could be related with electoral abstention. It is also explained how entropy is estimated. Section 3 presents the data used in this paper, while section 4 shows the results of the relationship between entropy and abstention. Section 5 concludes.

2. Entropy and electoral abstention

Entropy is a thermodynamics' concept, linked with the degree of disorder of a state. Created by Clausius in 1865, was defined by Ludwig Boltzmann as a probabilistic method to measure entropy. In the work that is the foundation of Information Theory, Shannon (1948) defines entropy as:

$$H(X) = -\sum p_i \log p_i \quad (1)$$

where p_i is the probability of the event i .

Shannon's entropy is a measure of uncertainty which is non-negative⁴. Zero is its minimum value, and it is just verified when there exists only one event, when there is certainty. The maximum entropy value is verified when the distribution is uniform ($p_i = 1/n_i$). Entropy measures total uncertainty of a not known probability distribution. In the scope of Information Theory, Shannon (1948) uses entropy as a measure of the amount of information transmitted by a message. Entropy could be also seen as a measure of randomness of a system.

Entropy for discrete distributions take its minimum when is certainty about an event. In terms of elections, it should occur if all electors vote in the same party (or candidate). The maximum

¹ Readers could also read MPRA working paper number 9234 (<http://mpa.ub.uni-muenchen.de/9234/>).

² Other methods of physical statistics were applied to election results, although with other objectives (see, for example, Fortunato and Castellano, 2007, or Filho *et al* 2003).

³ It is based on the interpretation of entropy as a measure of discontentment or disorder that Bailey (1990, 1993) uses measure in the context of Sociology.

⁴ Equation 1 refers to entropy for discrete distributions. Entropy for continuous distributions was also defined by Shannon (1948). It presents some different properties but it is not referred in this paper because it will not be used.

entropy value will occur when we are in the presence of a uniform probability distribution. Electorally speaking, it means that all parties (or candidates) have the same number of votes, result of their uncertainty.

This paper uses entropy for discrete distributions, to analyse the evolution of electoral results. Although its interpretation as uncertainty, in this particular case entropy is viewed as a measure of dissatisfaction of electors. In election results there are no probabilities to use to estimate entropy. However, elections' data are relative frequencies, which verify some of the main properties of probabilities: $p_i > 0$ and $\sum p_i = 1$. So, to estimate entropy we use vote proportions of each party. Suppose that in a given election there are three parties, with the following vector of vote proportions: $\{0,5; 0,3; 0,2\}$. In this case, the estimation of entropy following equation (1) would take the value of 1,0297. Because in this study the number of parties in the elections of different countries is different, it is calculated relative entropy,

$$HR = \frac{H(X)}{HM}, \quad (2)$$

where HM is the maximum entropy. Entropy takes its maximum value when we have a uniform distribution. Considering the same election when we have three parties and if vote proportions were $\{1/3; 1/3; 1/3\}$, entropy would take its maximum value, using equation 1, in 1,0986, and relative entropy will have the value of 0,9372.

There exist several studies trying to explain the reason for some electors vote and others abstain. Downs (1957), in a pioneer work in this research area, identifies the decision of vote with the fact that the elector has a greater benefit, when comparing with the cost of vote, in a similar decision when economic agents decide how to consume. One of the critics pointed to this analysis is the fact that it couldn't explain all what is behind the decision of an elector to vote. It happens because there are other kinds of motivations in the act of vote, like the civic duty sensation. Besides this, the decision of vote could be more related with an investment decision than with a consumption one, expecting a benefit in the future (see, in example, Riker and Ordeshook 1968, and Palfrey and Rosenthal 1983).

Other explanations for the decision to vote are made in models that use a spatial approach to explain that decision: the greater the proximity between elector's and candidates' opinion, the greater is the probability of an elector vote in that candidate. This type of analysis is done, for example, by Ferejohn and Fiorina (1974), Enelow and Hinich (1981), Paldam and Skott (1995), Kirchgässner (2003) and Gill (2005).

The purpose of this paper is not to make a model of the decision of vote or abstain but, as it was referred, to relate abstention with entropy, interpreting it as a measure of dissatisfaction. The question here is: should it be possible to use entropy as a measure of dissatisfaction of electors? In the particular case of USA elections, Gill (2005) refers that there exists a great complexity in the elections. It happens because candidates present a vague speech, letting some uncertainty in electors. Besides this, the lack of knowledge of electoral programs and the lack of time of electors to analyse those programs, could make a contribution to create that uncertainty. The existence of asymmetric information, according to the same author, could conduct some electors to think that the best alternative is to not vote (see also Enelow and Hinich 1981 and Feddersen and Pesendorfer 1997). So, and according to Gill (2005), if some electors vote and others do not, this is due to different levels of uncertainty of electors.

Paldam and Skott (1995) and Kirchgässner (2003) refer that political parties gain votes if, in the electors' view, they are visibly different between them. So, one possible explanation for abstention is the belief of electors that political parties have no qualitative different solutions, synonymous of dissatisfaction. Or, in the other hand, that political parties present promises that

they successively do not fulfil. Santo (2005) cites Georges Vedel affirming that one possible reason to the increase of abstention, in the case of France, is the depolitization, which can also be interpreted as a signal of population's dissatisfaction. Citing Freire and Magalhães, Santo (2005) refers that abstention could be understood as a temporary attitude of dissatisfaction in relation to the political system and inclusively as a lack of confidence⁵, the same motive indicated by Vargas (2002). The depolitization and lack of confidence, associated with the weakening of party identification, had been pointed by Abramson and Aldrich (1982) as possible explanations for the decrease of turnout, in the specific case of USA. The same authors point the declining beliefs about government responsiveness as another reason for the increase of abstention⁶. All these aspects could be also seen as dissatisfaction.

3. Data

Abstention is a phenomenon that occurs in all countries of the world and has been studied into several occasions and approaches. In this paper abstention is analysed for 16 European countries: Italy, Spain, Portugal, United Kingdom, the Netherlands, Germany, Belgium, France, Norway, Sweden, Finland, Greece, Switzerland, Austria, Ireland and Denmark. The study starts with elections after II World War. The main reasons for this choice are: (i) it is a mark on the world history; and (ii) data is more frequent from this date. Data was recovered in national websites⁷. Just parliamentary elections were considered. Following Santo (2005), the context of the election (if it is national, regional, European, presidential, etc.) could have influence in the level of abstention. So it is necessary to choose similar elections for countries.

In terms of the evolution of abstention, we found that the levels of abstention increased for almost all the countries studied in our data sample. This growing trend just not occurs in Spain, Norway and Denmark. In the cases of Germany, Belgium and Sweden, until half of sample, abstention had decreased. From that time to nowadays, they have the same increasing path⁸.

We use proportions of vote for the most voted parties to estimate discrete entropy. The number of parties used in each country, for the estimation of entropy, differs between countries but, in the same country, the number of parties is always the same. To make comparisons between different countries, and since entropy is sensitive in relation to the number of categories (parties) used for the analysis, it was calculated the relative entropy (HR) [see equation (2)]. The criterion used to choose the number of categories in each country is related with the number of parties most voted, in the first considered election. The objective was that, at the beginning of the sample, the proportion of the category "Others" shouldn't exceed 10%.

4. Results

Entropy is frequently used as a measure of uncertainty. In this paper it is used to analyse electoral processes, interpreting it as a measure of uncertainty of electors for politics in general. It could be considered as a measure of dissatisfaction and disillusion in relation to politics. As referred previously, electors' dissatisfaction is caused, according to some authors, by the

⁵ Bailey (1983) refers that maximum entropy is the "system death" (p. 114). The increase of entropy, in the context of elections and interpreted as social dissatisfaction shouldn't mean the "death of democracy" but should be analysed by politicians as a problem, like the increase of abstention.

⁶ Some studies try to explain abstention at an aggregate level. Blais *et al* (2003) try to identify the factors that have influence in the electoral participation, with GDP per capita showing a positive impact in participation. Also the dimension of population contributes to turnout, but in a negative form. Electoral systems that allow voting through alternative ways (mail, in advance or by proxy) and that have compulsory vote (penalizing those that do not vote) present also more relative participation in elections. Feddersen and Pesendorfer (1997) also identify the reduction of vote barriers as a potential variable which could raise turnout.

⁷ For some elections, in the absence of information about abstention, these data was recovered from www.idea.int/vt/.

⁸ Because of space constraints, source data and data of abstention along the time are not presented in this paper. It will be given by authors if requested.

weakening of the interest for politics and has a consequence on the decreasing of turnout, increasing abstention. In this sense, entropy should be positively correlated with abstention which means that, the greater the dissatisfaction of electors, abstention will take higher values (see, for example, Santo 2005 or Abramson and Aldrich 1982). Individual results, for each of sixteen countries in analysis, could be seen in table 1. Those values refer to the correlation between relative entropy and abstention, for each country.

Table 1. Individual results for correlation coefficient (R) between entropy and abstention^a.

Country	R
Italy	0,5927*
Spain	0,2031
Portugal	-0,5610
United Kingdom	0,5490*
Netherlands	0,1884
Germany	0,8072**
Belgium	0,1290
France	-0,5130*
Norway	0,5160*
Sweden	0,0390
Finland	-0,0279
Greece	0,3092
Switzerland	0,6859**
Austria	0,8397**
Ireland	0,7032**
Denmark	-0,3800

^a Individual results for correlation coefficient (R) between entropy and abstention. ** significant at 5% level. * significant at 1% level.

From the analysis of all countries in this database, just four have different correlation coefficients from expected: Portugal, France, Finland and Denmark. From these countries, just the French case is significant at 5% level. The coefficient of Portugal, besides taking a relatively high value (inclusively greater in absolute value when compared with the French coefficient), is not significant at 5%. In the Portuguese case this could be related with the fact that electoral files are often out of date.

For the countries that present positive coefficients, more than half are significant: Italy, United Kingdom and Norway, at 5% level and Germany, Switzerland, Austria and Ireland at 1% level. Our results show a strong support for what is proposed in this paper: abstention is related with uncertainty of electors, measured by entropy. Considering that any country has a sample smaller than twenty elections, results could be considered encouraging. Figure 1 shows the graphical representation of the different countries, considering these two variables. Relative entropy is presented in the horizontal axis and abstention in the vertical one.

If one of the problems could be the small dimension of the samples in some countries, it is interesting to verify what happens when we aggregate data from different countries, making just one sample. Because for the estimation of entropy were used different number of categories for countries, it was estimated a comparative measure. It was calculated the level of relative entropy ($HR = H / HM$), a measure that is greater than 0 and smaller than 1 and indicates the relationship of the value of entropy in each election and its maximum value, in the case of all categories have the same vote proportion. As the level of abstention is completely different for countries, it was done the calculus of abstention per capita, to make possible an international comparison between data⁹.

⁹ Abstention per capita represents the value of percentage of abstention for million inhabitants. A similar indicator to this is used in Strömberg (2004).

The sample recovered has a total of 270 elections. Estimating the correlation coefficient between relative entropy and the respective level of abstention per capita, the observed value is $0,2971$ and statistically significant. It is concluded that, besides results have some support for individual analysis, the same happens when aggregated data is considered.

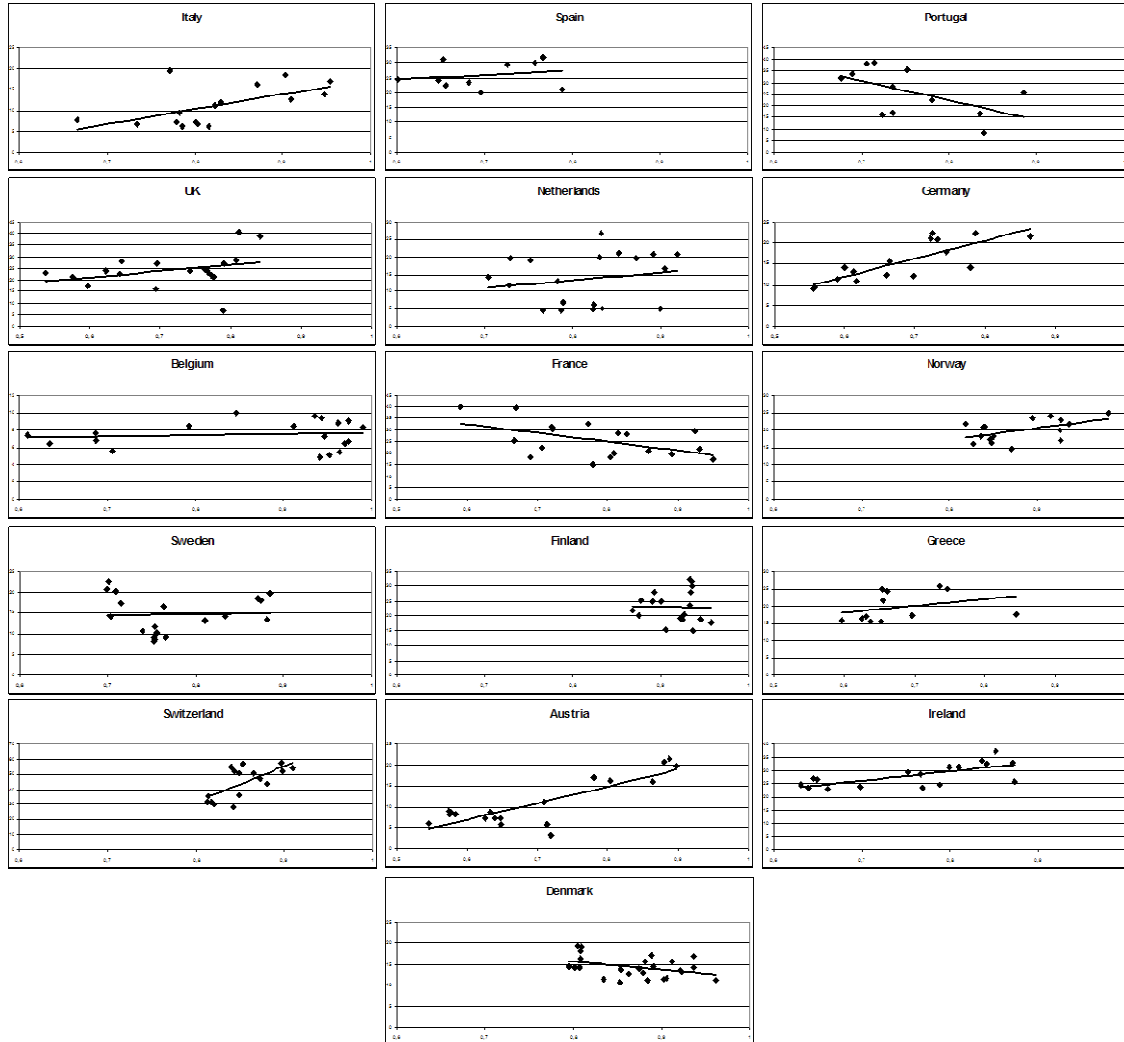


Figure 1. Relative entropy and abstention.

Entropy had been already used previously to analyse its relationship with turnout. Kirchgässner and Schimmelpfennig (1992) defend that, how much electors think that election's result is close, greater is their probability to decide to vote, because they consider their vote with capacity to be decisive. As elections with closer results mean higher levels of entropy, authors find evidence for United Kingdom and West Germany (elections in both countries in 1983 and 1987) for a positive relationship between entropy and turnout.

This paper presents entropy as a measure of dissatisfaction, positively correlated with abstention and, so, negatively correlated with turnout, contradicting previous results. A possible explanation could be related with the reduced number of elections used by Kirchgässner and Schimmelpfennig (1992), despite their use of information for several regional electoral circles. Another possible explanation is that entropy, as it was estimated, is a measure calculated after elections. The factors that can have influence in the decision of vote by electors, according to the authors, is their belief that elections will have close results, and these are data supplied not by final results but by pre-electoral opinion polls.

Coleman (2002, 2004) uses entropy as a measure of social conformity. Considering the turnout entropy, the author says that high conformity is associated with higher turnout (lower abstention) and lower values of entropy. Our results and respective interpretation are, somehow, in agreement with this author, since those point to the fact that higher entropy (in our case meaning dissatisfaction) is related with a higher abstention level.

Finally, it were compared three different measures normally used to measure dispersion and concentration: entropy, variance and Herfindahl index, all in relative terms¹⁰. The first two measures presented, between them, a close relation. When compared with abstention, both in absolute and per capita terms, relative entropy shows stronger results. Relative entropy shows an inverse relation, as expected, with the Herfindahl index. However, both measures present similar qualitative results when compared with abstention: how greater is abstention, it could be seen as dissatisfaction of populations but also has a positive correlation with electoral results (see table 2).

Table 2. Comparison of different measures: relative entropy (HR), relative variance (VARR), relative Herfindahl index (HHR) and abstention per capita (Abstpc)^b.

	HR	VARR	HHR	Abstpc
HR	1	0,6726**	-0,978**	0,2907**
VARR		1	-0,6175**	0,1535*
HHR			1	-0,2661**
Abstpc				1

^b Individual results for correlation coefficient (R) between entropy and abstention. ** significant at 5% level. * significant at 1% level.

5. Conclusion

This paper has the objective of extend the use of entropy in elections. Being entropy a measure of uncertainty, it can be seen as a measure of dissatisfaction of electors in relation to politics in general. We expect that when the dissatisfaction of electors is higher, the turnout will be lower and, consequently, abstention will be higher.

Our results point to the existence of a relation between abstention and entropy, which is positive for the large majority of the analysed countries. This may be a sign that, effectively, entropy could be seen as a dissatisfaction measure of electors, since it measures the dispersion of votes over the different political parties and at the same time, it is related with abstention. Besides that, the comparison between the dispersion and concentration measures (variance, entropy and Herfindahl index) shows that all present the same qualitative interpretation, but the relative entropy shows stronger results given its ability to capture this kind of phenomena.

It is also interesting to analyse the relationship between abstention and some economic and specific socio-political events as well as the extension of this analysis to other type of elections (for municipalities or presidential, for example). This is the main theme of a future research.

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¹⁰ Herfindahl index is given by $HH = -\sum p_i^2$ and relative Herfindahl index is given by $HHR = \left(HH - \frac{1}{N}\right) / \left(1 - \frac{1}{N}\right)$.

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