Bolsas de Integração na Investigação 2009/2010

Relatório Final

"Inventariação Bibliográfica dos Métodos de Extracção da Utilidade"

Sónia Isabel Caldeira Romero

Orientador: Prof. Doutor Luís Coelho
Co-orientador: Prof.ª Doutora Andreia Dionísio

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CEFAGE - Centro de Estudos e Formação Avançada em Gestão e Economia

Relatório referente à Bolsa de Integração na Investigação concedida pela Universidade de Évora - CEFAGE

"Inventariação Bibliográfica dos Métodos de Extracção da Utilidade"

Projecto de enquadramento: Exploração da Entropia em Modelos de Decisão.
Análise da Dependência Não-Linear, da Utilidade e Bem-Estar Económico e Social.
(PTDC/GES/70529/2006)
15 de Outubro 2009 - 29 de Outubro 2010

Trabalho realizado por:
Sónia Isabel Caldeira Romero
Curso: Gestão
Aluna n.º 21183

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Índice

1 - Introdução

O presente relatório visa descrever o modo como decorreu a bolsa de Bolsa de Integração na Investigação concedida, pela Universidade de Évora - CEFAGE (Centro de Estudos e Formação Avançada em Gestão e Economia), a Sónia Isabel Caldeira Romero, aluna da licenciatura em Gestão da referida universidade.

O tema, objecto de tratamento, da referida bolsa consiste numa “*Inventariação Bibliográfica dos Métodos de Extracção da Utilidade*” e enquadra-se no Projecto de enquadramento: Exploração da Entropia em Modelos de Decisão. Análise da Dependência Não-Linear, da Utilidade e Bem-Estar Económico e Social. (PTDC/GES/70529/2006).

A bolseira contou com o apoio e sugestões do orientador: o prof. Doutor Luís Coelho e com a co-orientadora: a prof. Doutora Andreia Dionísio. Ficou acordado que o *paper* resultante do trabalho efectuado fosse redigido na língua inglesa e portanto intitulado: “*Bibliographical inventorying of the elicitation utility´s methods*”.

A bolsa decorreu no período compreendido entre 15 de Outubro de 2009 e 29 de Outubro de 2010, estando agendada a apresentação oral do trabalho resultante da mesma para o último dia.

O relatório em causa tem, assim, como objectivo descrever as actividades realizadas pela bolseira durante os meses da referida bolsa e, dele fazem parte: a metodologia seguida, os resultados obtidos, a discussão dos mesmos e as referências consultadas, sendo que, no anexo, se encontra o *paper* referido.

2 - Metodologia

A metodologia seguida para elaborar o *paper* em causa encontra-se indicada no cronograma que se encontra na página 3 (tabela 2.1). Com vista à elaboração do mesmo foram consultadas várias fontes bibliográficas, tendo havido o recurso a pesquisas nos sites www.b-on.pt, www.google.com tendo sido ainda consultado o trabalho: "Decisão em contexto de incerteza: o comportamento dos agricultores na região de sequeiro do Alentejo face à reforma intercalar da política agrícola comum", da autoria do prof. Doutor Luís Coelho.

Com base no cronograma referido neste relatório, pode verificar-se que os primeiros cinco meses da bolsa foram dedicados a pesquisa bibliográfica, tendo sido criado um registo, sob a forma de uma tabela, na qual constavam os seguintes itens: data, autores, revista, nome do artigo, método utilizado, objectivos e principais conclusões. Assim, de cada artigo consultado era extraída este tipo de informação sendo que este tipo de procedimento se revelou mais eficaz para proceder à inventariação bibliográfica.

Entre Fevereiro e Maio de 2010 a bolseira dedicou-se ao estudo da utilidade e à actualização da tabela referida.

Nos meses de Junho e Julho do mesmo ano foi efectuado um relatório parcial e, em Agosto e Setembro teve lugar a recolha complementar de informações e a elaboração do relatório final.

O último mês da bolsa caracterizou-se pela entrega do presente relatório e do *paper* resultante do trabalho desenvolvido, estando a apresentação do mesmo agendada para o dia 29 de Outubro de 2010.

Durante o período de realização da bolsa verificou-se o acompanhamento, sob a forma de reuniões periódicas, por parte do orientador e da co-orientadora tendo ambos cedido e sugerido bibliografia a consultar bem como efectuado sugestões de melhoria do *paper* apresentado. Nas referidas reuniões foram também abordadas questões sobre metodologia de pesquisa, tendo sido promovida uma troca de informações entre os orientadores e a bolseira. No esquema da figura 2.1 encontram-se as etapas ilustrativas da metodologia seguida.

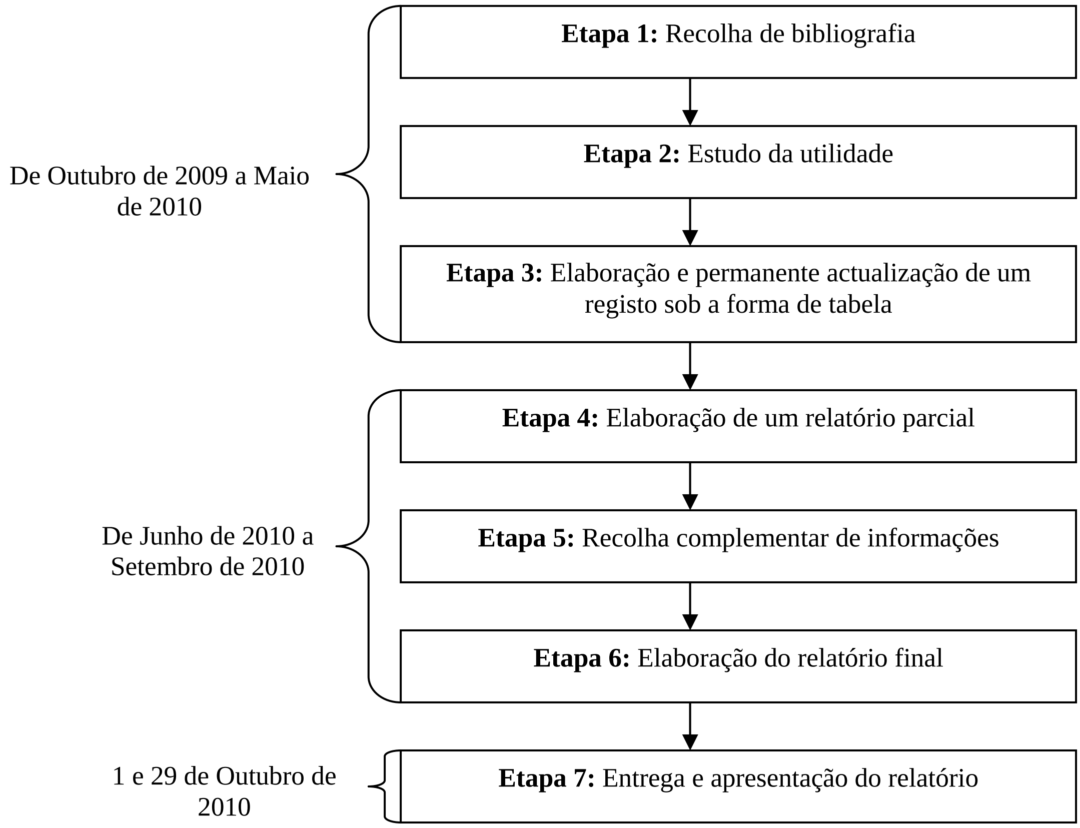

Figura 2.1 - Etapas da metodologia seguida.

A tabela 2.1 refere-se ao cronograma referido.

Tabela 2.1 - Cronograma.

	Out. 2009	Nov. 2009	Dez. 2009	Jan. 2010	Fev. 2010	Mar. 2010	Abr. 2010	Mai. 2010	Jun. 2010	Jul. 2010	Ago. 2010	Set. 2010	Out. 2010
Etapa 1	■	■	■	■	■								
Etapa 2					■	■	■	■					
Etapa 3					■	■	■	■					
Etapa 4									■	■			
Etapa 5											■	■	
Etapa 6											■	■	
Etapa 7													■

3 - Resultados

A pesquisa das fontes bibliográficas referidas permite afirmar que existe bastante informação, a maior parte dela dispersa, relativa, não só à utilidade em geral mas também relativa aos métodos de extracção da utilidade.

Uma das primeiras obras encontradas referente aos métodos citados remonta ao ano de 1967, ano este em que Fishburn referiu a existência de 24 métodos para estimar a utilidade: *Ranking*, *Direct Rating*, *Standard Gamble 1*, *Standard Gamble 2*, *Direct Midpoint (Bisection)*, *Probabilistic Midpoint*, *Direct Ordered Metric*, *Prob. Ordered Metric*, *Ranking*, *Direct Rating*, *Probabilistic Rating*, *Successive Comparison*, *Half-Value Sum*, *Direct Ordered Metric*, *Ordered Metric*, *Prob. Ordered Metric*, *Single Trade-off*, *Double Trade-off*, *Single Transformation*, *Double Transformation*, *Discrete Trade-off*, *Discrete Transformation*, *Discrete Adjacency* e *Saw-Tooth*.

Em 1982 foi a vez de Hershey considerar 4 métodos para construir funções de utilidade von Neumann-Morgenstern:

- O *Certainty equivalence method*,
- O *Probability equivalence method*,
- O *Gain equivalence method*,
- O *Loss equivalence method*.

Foi ainda possível obter informação acerca de Farquhar (1984) que considerou 4 novas categorias:

- *Preference comparison methods*,
- *Probability equivalence methods*,
- *Value equivalence methods*,
- *Certainty equivalence methods*.

Um dos trabalhos consultados e também dos mais recentes foi o de Coelho (2005) que concluiu acerca da existência de 4 métodos para extrair as preferências:

- *Direct scaling methods*,
- *Certainty equivalent method*,
- *Probability equivalent method*,
- *Equivalent risk´s methods*.

4 - Discussão

Apesar de existirem várias definições, mais ou menos complexas, a utilidade pode ser definida como uma medida da satisfação do decisor.

Constatou-se a dedicação a esta temática por parte de vários investigadores, nomeadamente: Robbins (1932), Hicks e Allen (1934), Stigler (1950), Blaug (1962), von Praag (1968), Loomes and Sugden (1982), Broome (1991), Tinbergen (1991), Kahneman (1994), Kapteyn (1994), Gilboa e Schmeidler (2001), Robson (2001), Frey e Stutzer (2002), Ferrer-i-Carbonell (2004).

Foram referidos, ao longo do tempo (desde 1967 a 2006), vários métodos de extracção da utilidade.

O *standard gamble method* e o *time trade-off method* são, segundo Osch *et al* (2004), dois dos métodos mais usados para medir a utilidade.

Foram ainda discutidas e comparadas as vantagens e desvantagens subjacentes aos 4 métodos passíveis de serem utilizados para extrair as preferências - sugeridos por Coelho (2005) - tendo-se concluído que aplicando os diferentes métodos, obtêm-se resultados diferentes.

Wakker e Deneffe (1996) sugeriram um método alternativo: o *lottery-equivalent method*.

Mais recentemente Villas´is *et al* (2006) discutiu a possibilidade de se alterarem os métodos de extracção da utilidade, adoptando um novo método, o: *equal tails*.

Formularam-se ainda conclusões referentes a duas áreas em que é possível a aplicação dos métodos abordados: a agricultura e a medicina.

5 - Bibliografia

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Anexo

Paper: BIBLIOGRAPHICAL INVENTORYING OF THE ELICITATION UTILITY´S METHODS

BIBLIOGRAPHICAL INVENTORYING OF THE ELICITATION UTILITY´S METHODS

Sónia I. C. Romero
romero-1001@hotmail.com
CEFAGE - Centro de Estudos e Formação Avançada em Gestão e Economia
Universidade de Évora
7000-803 Évora

Abstract

It addresses to some of the various definitions of utility, over time, emphasis being given to elicitation utility´s methods. Are also mentioned some of the advantages and disadvantages inherent to each method, and list some of the possible applications of these methods, particularly in areas such as agriculture and medicine. To obtain this information were consulted several papers of scientific nature. As a result it should be noted that exists several elicitation utility´s methods, proposed by Fishburn, Hershey, Farquhar, Coelho and Villas´is.

Keywords: Adjacent gambles, Anchored gambles, Assorted gambles, Balanced values, Block methods, Bottom-up method, Bounding technique, Certainty equivalence method, Chaining methods, Convergence technique, Direct estimation technique, Direct midpoint (Bisection), Direct ordered metric, Direct rating, Direct scaling methods, Discrete adjacency, Discrete trade-off, Double trade-off, Double transformation, Elicitation utility´s methods, ELCE (Equally Likely Certainty Equivalent), ELRO, Equisection, Equivalent probability method, Equivalent risk methods, Expected utility, Extreme gambles, Fractile, Fractionation, Gain equivalence method, Half-value sum, Hybrid methods, Indifference points´s estimation, Interlocking extreme gambles, Loss equivalence method, Midpoint chaning, Multiplication, Paired-gamble methods, Prob. ordered metric, Preference comparison methods; Probability equivalence method, Probabilistic midpoint, Probabilistic rating, Ranking, Saw-tooh, Single trade-off, Single transformation, Standard gamble 1, Standard gamble 2, Successive comparison, Top-down method, “Trade-off” method, Uniform sequence, Utility, Value equivalence methods

1 - Introduction

Over time the utility´s concept has met several definitions, more or less complete. Will have been in the year 1738 that was made one of the earliest references to the utility. The publication of an essay by Daniel Bernoulli, in the “Papers of the Impirial Academy of Sciences in St. Petersburg” and that he considered that the value of an item should be based on the utility that provided in detriment of the respective price.

This paper is intended as a bibliographical inventorying of the elicitation utility´s methods.

The objectives of this paper are therefore:

- To refer some utility´s definitions;
- To gather a collection of information about elicitation utility´s methods;
- To refer the main advantages and disadvantages of each method;
- To mention some of the applications of elicitation utility´s methods;

In order to achieve these objectives, the methodology consisted of the research and analysis of numerous scientific papers related to the topic in question.

This paper is organized as follows. Section 2 refers to the utility concept. Section 3 deals with elicitation utility´s methods. Section 4 refers to methods´s applications and section 5 concludes the paper.

2 - The utility concept

Bentham (1789) set the utility concept as that "point out what we ought to do, as well as determine what we shall do". This usage was retained in the economic writings of the 19th century, but it was gradually replaced by a different interpretation (Kahneman *et al*, 1997). According to Kahneman *et al* (1997), in current economics and in decision theory, the utility of outcomes and attributes refers to their weight in decisions: utility is inferred from observed choices and is in turn used to explain these choices. Since the beginning of the 20th century, after what has since become known as the ordinal revolution, utility has been taken as an ordinal concept, based solely on observable choice, in mainstream economics (Pareto, 1906). Robbins (1932), Hicks and Allen (1934), Stigler (1950), Blaug (1962), van Praag (1968), Loomes and Sugden (1982), Broome (1991), Tinbergen (1991), Kahneman (1994), Kapteyn (1994), Gilboa and Schmeidler (2001), Robson (2001), Frey and Stutzer (2002), Ferrer-i-Carbonell (2004) and others are some examples of researchers who have studied the utility.

In a very brief one can say that the utility measures the satisfaction of the decison maker and, according to some researchers, consider the measurability of welfare, the satisfaction and utility as synonyms.

The calculation of the utility function and the definition of probabilities are characterized by subjectivity. When we determine the utility function we are building a mathematical model that represents the preferences (Coelho, 2005). The extraction of preferences is accomplished with the use of a set of questionnaires (these are made in laboratories, making use of interviews with MBA students who are not directly related to economic problems) that by applying some methods allows to obtain the utility function (Coelho, 2005).

According to Ottoy (1992), utility theory is an analytical method for making a decision concerning an action to take, given a set of multiple criteria upon which the decision is to be based.

Expected utility (EU) is the standard normative model for decision under risk and uncertainty - in EU one determines quantitative values, “utilities”, for outcomes, and probabilities for uncertainties (Wakker and Stiggelbout, 1993).

Utility functions are an important component of normative decision analysis, in that they characterize the nature of people´s risk - taking attitudes (Hershey *et al*, 1982).

It was also proposed by Bernoulli and Cramer, an expression of cardinal measure of utility that can be used as a means of evaluating the various choices or alternatives of a decision problem. The expression relates the utility function with the initial wealth, the probabilities and outcomes, as follows:

$$V(y) = \sum_{i=1}^{n} u(w_o + X_i).\, p(X_i) \qquad (2.1)$$

Where:

V - Value of the alternative;

y - Alternative;

u - The utility function;

w_o - Initial wealth;

p - Probabilities;

X_i - Results.

According to Coelho (2005), the utility function can be used as a method of evaluating the decision in risk context. The same source also mentions that for a discrete outcome vector of dimension n, the expected utility is evaluated as follows for different discrete alternatives of a problem:

$$EU(y) = \sum_{i=1}^{n} p_i .u(w_o + X_i) \qquad (2.2)$$

Where:

EU - Expected utility;

y - Alternative;

u - The utility function;

w_o - Initial wealth;

X_i - Results;

p - Probabilities.

Several studies have shown that utility measurement based on expected utility leads to inconsistencies. According to Coelho (2005), it is generally accepted as a normative model of rational choices and is extensively applied as a descriptive model of economic behavior. In expected utility theory assumes that individuals are faced with the probability distributions or they can define subjective probabilities for all relevant events (Coelho, 2005).

3 - Elicitation utility´s methods

One of the earliest references about the elicitation utility´s methods was found in the 60´s and more specifically in the work of Fishburn (1967), entitled "Methods of estimating additive utilities". In it, Fishburn stated twenty-four methods to estimating additive utilities. Table 3.1 relates to the methods of estimating additive utilities considered by Fishburn.

Table 3.1 - Methods of estimating additive utilities (Fishburn, 1967).

Group	*Method*	*Classification*
I	1 - *Ranking*	- The method does not use probabilities; - The method is based on preference judgments (the method is based on "direct" inequality judgments on utilities); - The method is best used with discrete factors.
	2 - *Direct Rating*	- The method does not use probabilities; - The method is based on "direct" equality judgments on utilities; - The method is usable with either discrete or continuous factors.
	3 - *Standard Gamble 1*	- The method uses probabilities in scaling utilities; - The method is based on indifference judgments; - The method is usable with either discrete or continuous factors.
	4 - *Standard Gamble 2*	- The method uses probabilities in scaling utilities; - The method is based on indifference judgments; - The method is best used with continuous factors.
	5 - *Direct Midpoint (Bisection)*	- The method does not use probabilities; - The method is based on "direct" equality judgments on utilities; - The method is best used with continuous factors.

	6 - *Probabilistic Midpoint*	- The method uses only the probability 1/2 in scaling utilities; - The method is based on indifference judgments; - The method is best used with continuous factors.
	7 - *Direct Ordered Metric* (1)	- The method does not use probabilities; - The method is based on "direct" inequality judgments on utilities; - The method is best used with discrete factors.
	8 - *Prob. Ordered Metric* (1)	- The method uses only the probability 1/2 in scaling utilities; - The method is based on preference judgments; - The method is best used with discrete factors.
II	9 - *Ranking*	- The method does not use probabilities; - The method is based on preference judgments (the method is based on "direct" inequality judgments on utilities); - The method is most applicable with binary (two-level) factors for estimating the v_i in (2**) or is applicable for estimating the scale-transformation parameters in (2*).
	10 - *Direct Rating*	- The method does not use probabilities; - The method is based on "direct" equality judgments on utilities; - The method is most applicable with binary (two-level) factors for estimating the v_i in (2**) or is applicable for estimating the scale-transformation parameters in (2*).
	11 - *Probabilistic Rating*	- The method uses probabilities in scaling utilities; - The method is based on indifference judgments; - The method is most applicable with binary (two-level) factors for estimating the v_i in (2**) or is applicable for estimating the scale-transformation parameters in (2*).
	12 - *Successive Comparison* (9)	- The method does not use probabilities; - The method is based on preference judgments (the method is based on "direct" inequality judgments on utilities; - The method is most applicable with binary (two-level) factors for estimating the v_i in (2**) or is applicable for estimating the scale-transformation parameters in (2*).
	13 - *Half-Value Sum* (9)	- The method does not use probabilities; - The method is based on indifference judgments (the method is based on "direct" equality judgments on utilities); - The method is most applicable with binary (two-level) factors for estimating the v_i in (2**) or is applicable for estimating the scale-transformation parameters in (2*).

	14 - *Direct Ordered Metric* (9)	- The method does not use probabilities; - The method is based on "direct" inequality judgments on utilities; - The method is most applicable with binary (two-level) factors for estimating the v_i in (2**) or is applicable for estimating the scale-transformation parameters in (2*).
	15 - *Ordered Metric* (9)	- The method does not use probabilities; - The method is based on preference judgments (the method is based on "direct" inequality judgments on utilities); - The method is most applicable with binary (two-level) factors for estimating the v_i in (2**) or is applicable for estimating the scale-transformation parameters in (2*).
	16 - *Prob. Ordered Metric* (9)	- The method uses only the probability ½ in scaling utilities; - The method is based on preference judgments; - The method is most applicable with binary (two-level) factors for estimating the v_i in (2**) or is applicable for estimating the scale-transformation parameters in (2*).
III	17 - *Single Trade-off* 18 - *Double Trade-off* 19 - *Single Transformation* 20 - *Double Transformation*	- The method does not use probabilities; - The method is based on indifference judgments (the method is based on "direct" equality judgments on utilities); - The method is best used with continuous factors.
IV	21 - *Discrete Trade-off* 22 - *Discrete Transformation* 23 - *Discrete Adjacency* (1)	- The method does not use probabilities; - The method is based on indifference judgments (the method is based on "direct" equality judgments on utilities); - The method is applicable using one discrete and one continuous factor or one continuous and another factor: the first letter tells which factor is receiving "new" utility information from the judgment.
V	24 - *Saw-Tooth*	

(2*) $u(X_1, X_2, \ldots, X_n) = v_1.w_1(X_1) + v_2.w_2(X_2) + \ldots + v_n.w_n(X_n)$

(2**) $u(X_1, X_2, \ldots, X_n) = X_1.v_1 + X_2.v_2 + \ldots + X_n.\, v_n$

In table 3.1 it is possible to verify the presence of five groups. According to Fishburn (1967) group I´s methods are those that involve only one factor at a time. In turn the groups II to V are considered different levels of more than one factor in each judgment. The number in parentheses following the name of a method identifies another method that is a prerequisite of the named method (Fishburn, 1967). It should be noted, for example, that the method 1 is a prerequisite of the 7, 8 and 23 methods.

Later, in 1982, Hershey *et al* stated in their work: “Sources of BIAS in assessment procedures for utility functions”, the existence of four different methods for constructing von Neumann-Morgenstern utility functions, namely:

1 - The *certainty equivalence (CE) method*;

2 - The *probability equivalence (PE) method*;

3 - The *gain equivalence (GE) method*;

4 - The *loss equivalence (LE) method*.

In 1984, Farquhar, with regard to the “utility assessment methods” consider four new categories:

1 - *Preference comparison methods*;

2 - *Probability equivalence methods*;

3 - *Value equivalence methods*, and

4 - *Certainty equivalence methods*.

Apart from these, the author also mention:

- *Hybrid methods*,
- *Paired-gamble methods* and other approaches.

Here is the description of the four categories.

1 - *Preference comparison methods*

In a *preference comparison* between the gamble (x, α, y) and the sure outcome w, an individual specifies the relation R (either >, <, or ~) such that the expression (x, α, y) $\underline{R}$ w holds (Farquhar, 1984). According to Farquhar (1984), preference comparison methods involve a sequence of such comparisons, (x_1, α_i, y_i) $\underline{R_1}$ w_1 for i = 1, 2, …, n, where the probabilities, values and standards are chosen in particular ways. This type of methods are used to investigate the attitudes towards risk, to verify the consistency of utility.

2 - *Indifference points´s estimation*

According to Farquhar (1984), the estimation of an indifference point (either a probability, value, or standard) can be accomplished in several ways.

a) The *direct estimation technique* considers an individual´s indifference point;

b) The *convergence technique* proceeds to the adjustment of points of preference until the decision maker expresses indifference;

c) The *bounding technique* considers upper and lower limits and a range of responses. These limits are being changed even if they manage preferences.

3 - *Probability equivalence methods*

Probability equivalence methods require that an individual specify an indifference probability α for which (x, α, y) ~ w, where w is between x and y (Farquhar, 1984). In this method, two points are chosen: x_o and x_{n+1} in X, where $x_o < x_{n+1}$ (the points may be the best and worst outcome in X). It is intended for the assess the utilities of the points $x_o < x_1 < \ldots < x_{n+1}$ using to this effect one of the following methods:

a) *Extreme gambles:*

In this method the reference points of X will be the extremes of all gamble. This method is easy to use but there are susceptibilities to dependence in the responses.

b) *Adjacent gambles:*

This method uses gambles over the "locally best and worst" values for each x_i (Farquhar, 1984). There will be one equation of the type:

$$u(x_i) = \alpha_i . u(x_{i+1}) + (1 - \alpha_i) . u(x_i - 1) \tag{3.1}$$

to each individual´s n responses.

One advantage of this method (compared to the earlier) is that the individual is not asked to assess probabilities near 0 or 1.

c) *Assorted gambles:*

In contrast to the previous methods, this method may require the numerical solution of n or more equations to determine $u(x_1)$, …, $u(x_n)$ (Farquhar, 1984).

4 - *Value equivalence methods*

This methods presupposes the existence of an indifference value x such that (x, α, y) ~ w. These methods assume a continuum of values in X so that an x exists satisfying this indifference relation (Farquhar, 1984). Here are some value equivalence methods:

a) *Uniform sequence*

In this case is built a scale of equally-spaced values using, for this purpose, an uniform sequence approach. According to Farquhar (1984) there are two basic variations of this approach for gamble comparisons:

- The *Bottom-up method*,
- The *Top-down method*.

In the bottom-up method x_0 and x_1 are fixed and $x_0 < x_1$. Are obtained additional values from (x_{i+1}, x_{i-1}) ~ x_i, for i = 1, …, n.

In the top-down method x_0 and x_1 are fixed and $x_1 < x_0$. These methods seem most appropriate for unidirectional attributes, like media exposure or waiting time, that are bounded from below or above in utility (Farquhar, 1984).

b) *Balanced values*

Assuming that {x_0, x_1, … x_n} is an uniform sequence anchored at a "neutral value" x_0; for definiteness, let $u(x_i) = i$ for i = 0, 1, …, n. This method can be used to derive $u(x_{i'}) = - i$. Similarly, if the uniform sequence {x_0, x_1, …, x_n}has $u(x_i) = - i$, then the method of balanced values yields $u(x_{i'}) = i$ (Farquhar, 1984). It is not necessary to consider that {x_0, x_1, …, x_n} is an uniform sequence in order to implement this method.

c) *Multiplication*

According to Farquhar (1984), the multiplication method for comparing gambles is related to a ratio scaling procedure suggested by Galanter (1962) and to the fractionation methods described in Torgerson (1958). Considering as reference points x_0 and x_1, where $u(x_0) \equiv 0$. In this method can be used probabilities α_i instead of α being that, normally, the considered probability is 1/2.

d) *Equisection*

In this method is ask to an individual to divide the interval x_0 to x_{n+1} in n identical sections.

In general we can say that value equivalence methods seem to be satisfactorily for assessing utility functions, it is still needed to determine, through research, its strength and weaknesse. The methods are simple analytically, but the use of chained responses in the sequences of gamble comparisons may have undesirable effects in some applications (Farquhar, 1984).

5 - *Certainty equivalence methods*

In this type of methods it is ask to an individual to "specify a sure outcome w": the certainty equivalent, for which (x, α, y) ~ w. Here are some certainty equivalence methods:

a) *Fractile*

This is a method that is easy to implement where utility calculations are fast. This method is similar to the extreme gambles method of probability equivalence and, for this reason, both have the same advantages and disadvantages. The fractile method often takes the endpoints of X as x_0 and x_*, because any points below x_0 or above x_* need to be determined by another assessment procedure (Farquhar, 1984).

b) *Chaining methods: fractionation and midpoints*

According to Farquhar (1984), like a few other methods, chaining methods allow one to assess additional values one at a time until either enough points are available to estimate the utility function satisfactorily or the assessment process must be terminated for some reason.

Due to his simplicity, the certainty equivalence methods have been widely used.

Farquhar (1984) also considers the existence of:
- *Hybrid assessment methods* and
- *Paired-gamble methods*.

- *Hybrid assessment methods*

"There are many possibilities for merging basic assessment methods to procedure hybrid methods."

- *Paired-gamble methods*

In this type of method are considered the following methods:

a) *Preference comparison*
b) *Probability equivalence*

a) Preference comparison

This method is characterized for constructing an ordered metric scale for utilities.

b) Probability equivalence

Considers various possibilities, namely using paired-gamble comparisons. For example:

. Extreme gamble;

. Adjacent gambles;

. Anchored gambles.

c) *Value equivalence*

Another approach in developing paired-gamble methods using value equivalence is to have common values, $y_i \equiv z_i \equiv x_0$ (Farquhar, 1984).

Table 3.2 summarizes the utility assessment methods described in Farquhar´s work.

Table 3.2 - Utility assessment methods (Farquhar, 1984).

I - Standard - gamble methods
1. Preference comparison methods: $(x_1, \alpha_1, y_1)\ \underline{R_1}\ .\ w_1$
2. Probability equivalence methods:
(a) Extreme gambles: $(x_{n+1}, \underline{\alpha_1}, x_0) \sim x_1$
(b) Adjacent gambles: $(x_{l+1}, \underline{\alpha_l}, x_{l-1}) \sim x_l$
(Assorted gambles) $(x_k, \alpha_l, x_{j1}) \sim x_l$, where jl < i <ki
(c) Anchored gambles: $(x_{l+1}, \underline{\alpha_l}, x_0) \sim x_l$
3. Value equivalence methods:
(a) Uniform sequence: $(x_{l+1}, x_{l-1}) \sim x_l$
(b) Balanced values: $(\underline{x_{l'}}, x_l) \sim x_o$, for known $u(x_0)$ and $u(x_l)$
(c) Multiplication: $(x_{l+1}, \alpha, x_0) \sim x_l$
(d) Equisection : find $x_l, \ldots, x_n$ such that $(x_{l+1}, x_{i-1}) \sim x_l$ (bisection, trisection, quadrisection, and n-section)
4. Certainty equivalence methods:
(a) Fractile: $(x_*, \alpha_l, x_0) \sim x_l$ (chaining methods)
(b) Fractionation: $(x_l, \alpha, x_0) \sim \underline{x_{l+1}}$
(c) Midpoint chaning: $(x_*, x_0) \sim \underline{x_l}$, $(x_l, x_0) \sim \underline{x_2}$, $(x_*, x_l) \sim \underline{x_3}$
II. Paired-gamble methods
5. Preference comparison methods: $(x_l, \alpha_l, y_l)\ \underline{R_i}\ (w_i, \beta_l, z_l)$
6. Probability equivalence methods: $(x_l, \alpha_l, y_i) \sim (w_l, \beta_1, z_l)$ where both w_i and z_l are between x_l, and y_l
(a) Extreme gambles: $(x_{n+1}, \underline{\alpha_1}, x_0) \sim (x_{l+1}, x_{l-1})$
(b) Adjacent gambles: $(x_{l+2}, \underline{\alpha_l}, x_{i-2}) \sim (x_{l+1}, x_{l-1})$
(c) Anchored gambles: $(x_{i+1}, \underline{\alpha_l}, x_0) \sim (x_l, x_0)$
(d) Interlocking extreme gambles: $(x_{n+1}, \underline{\alpha_l}, x_0) \sim (x_{l+1}, \underline{\alpha_l}, x_{l-1})$
(e) Interlocking adjacent gambles: $(x_{n+2}, \underline{\alpha_l}, x_{l-2}) \sim (x_{l+1}, \underline{\alpha_i}, x_{l-1})$
7. Value equivalence methods: $(x_i, \alpha_l, y_l) \sim (w_l, \beta_l, z_i)$

(a) Equal differences: $(\underline{x_i}, y_l) \sim (w_i, z_i)$
(b) Anchored values: $(\underline{x_{l+1}}, \alpha, x_0) \sim (x_i, \beta, x_0)$ (multiplication for $\alpha < \beta$; fractionation for $\alpha < \beta$)
III. Hybrid methods
8. "Block" methods: Modified midpoint chaining method; modified variable range gamble method
IV. Other methods
9. Local coherence methods: $(x, \underline{\alpha}, y) \leftrightarrow (x, \beta_l(\alpha), y) \sim (w, \beta_2(\alpha), y)$

The researchers who are dedicated to obtaining preference elicitation´s methods have concluded about the various factors influencing the methods in question, including proposing modifications to these methods (Coelho, 2005). Hershey, Kunreuther and Schoemaker (1982) had identified five factors that influence the extraction of preferences: (i) the method used, (ii) the values of the outcomes and probabilities, and (iii) the use of gains, losses or the combination of two, (iv) the form how the risk is presented, and (v) the context of the decision.

In the year 2005, Coelho has concluded about the existence of four methods for direct extraction of preferences:

- Direct scaling methods,
- Certainty equivalent method,
- Probability equivalent method,
- Equivalent risk´s methods.

The following is a description of such methods.

3.1 - Direct scaling methods

According to Coelho (2005), in the methods of direct scaling the decision maker is asked to assess directly the results on a numerical scale. There is, for this purpose, a ranking scale that measures satisfaction. Thus, for example, is request that the individual decision maker assign the value 0 to what it considers the worst outcome and the value 100 to the more favorable outcome. A value of 50 to an outcome that lies halfway between the best and the worst outcome, etc (Wakker and Deneffe, 1996).

Among the advantages behind this method are noted as follows:

- It´s easy to use;
- The fact that there are no problems with the distortion of probabilities;

- The fact that it can be used to elicit the utility function of Generalized Expected Utility models.

According to Coelho (2005) the fundamental problem of this method is that the use of the direct scaling to extract utilities like von Neumann and Morgenstern needs theoretical justification.

3.2 - The ELCE (Equally Likely Certainty Equivalent) method/ Certainty equivalent methods

The ELCE method is widely used to elicit the von Neumann-Morgenstern´s utility function.

Certainty equivalent methods generally yield greater risk-seeking than probability equivalence methods (Hershey *et al*, 1982). According to Coelho (2005) this method can be defined as the amount that makes the decision maker indifferent between one alternative with a risk and a certain amount. This method is based on expected utility theory and it is ask to the decision maker to make the comparison of the alternative (x_1, p; x_2, 1-p) obtaining a correct result. The analyst then varies the right result until the decision maker reveal indifference between this and the alternative proposal, obtaining the following equality (Coelho, 2005):

$$u(CE) = p.u(X_1) + (1-p).u(X_2) \quad (3.2)$$

Where:
CE - Certainty equivalent,
u - Utility function,
p - Probabilities,
X_1 e X_2 - Results.

Thus to elicit utilities it is necessary (Wakker and Deneffe, 1996):
1.º - To consider two outcomes (x_2 and x_1 in which $x_2 > x_1$), fixed such that the range of outcomes between them includes all outcomes of interest;
2.º - To associate two values (eg.: $u(x_1) = 0$ and $u(x_2) = 1$) and ask the decision maker to establish the certainty equivalent that makes him indifferent to (x_1, p; x_2, 1-p);
3.º - To elaborate two alternatives that have the certainty equivalent already established and the consequences are more favorable and less favorable, and, then request to the decision maker to establish a certainty equivalent;

4.º - Construct two new games with the same probabilities, with the certainty equivalent already established and the best and worst outcomes;
5.º - Asks to the decision maker to establish the certainty equivalent of these games;
6.º - This procedure is adopted there until the necessary points exist, in the curve of the certainty equivalent, to obtain the expected utility function.

According to Coelho (2005), the number of questions depends on the degree of accuracy deemed appropriate, the degree of consistency of the questions made and the decision maker´s degree of impatience. The same source also states that, after obtaining a enough number of questions of validation will have to be accomplished, such as:

$$(EC_2, p; EC_3; 1\text{-}p) = (EC_5, p; EC_6; 1\text{-}p) = (EC_4, p; EC_7; 1\text{-}p) = EC_1.$$

Wakker and Deneffe (1996) also says that the CE method is often used in a bisection form, that only uses probabilities 1/2 and that:

- an outcome denoted CE´(1/4) is obtained through an indifference CE´(1/4) ~ (0, 1/2; CE(1/2));
- substitution of EU shows that the utility of CE´(1/4) must be 1/4.

The biggest disadvantage of the method in question is that to determine the utility function is necessary to know the probabilities (Coelho, 2005). The probabilities vary according to use the author considered. Thus:

- Anderson, Dillon and Hardaker (1977) consider the 50:50;
- Kahneman and Tversky (1929) disagree from the opinion of the previous authors also considering that risk aversion is overrated;
- Wakker and Deneffe (1996) consider overvalued probabilities < 1/3, understated the probabilities > 1/3 and non transformed probabilities = 1/3;
- Tversky and Fox (1995) consider 1/3 as being the probability more adjusted to extract utilities. However, the use of probabilities of 1/3 does not eliminate the distortions caused by the extraction of the utility function by the certainty equivalent method (Coelho, 2005).

The fact that it is necessary to calculate the expected value is another disadvantage of this method. When applied to decision-makers who dislike the game, can lead to distortions in the responses. Answers provided by the decision maker are used to “cascade”.

As mentioned advantage is the fact that it is possible for the analyst to control the process. It should also be noted that a new version on this method is given to the alternative decision-maker (x_1, p; x_2, 1-p) being him, according to Coelho (2005), request to fix a sum insured (certainty equivalent) of so that it is indifferent to choose between the two. By altering the probability p is possible to obtain the curve of utility.

3.3 - Probability equivalent method

Another approach of the utility function is through the probability equivalent method that is to adjust the probabilities with reference to one of the alternatives in order to find the indifference between this alternative and a certain outcome (Coelho, 2005). In this method the decision maker´s being asked to make the comparison of the alternative (x_1, p; x_2, 1-p) with a certain result x_{EC} ($x_1 < x_{EC} < x_2$). The analyst will varies the probabilities until the decision maker reveals indifference in relation to alternative and the rignt result $x_{EC.}$

Like the certainty equivalent method are considered a minimum outcome (x_1) and a maximum outcome (x_2). The utility function is obtained through various p values, which are resulting in several changes of x_{EC}. The method in question has the same disadvantages as the certainty equivalent method.

According to Coelho (2005), this method is more susceptible to suffer from answer effects, because while in the questions of the certainty equivalent method it is requested to the decision makers to respond in terms of results, in the questions of the probability equivalent method is required that they answer in terms of probabilities.

3.4 - Equivalent risk methods

In this type of methods the decision makers establish a comparison between two alternatives where risk exists and, according to Coelho (2005), they replace one of the outcomes or the probabilities to obtain the indifference. Thus, according to the same source, rather than to seek the indifference between a certain outcome and an alternative to risk, they looks the indifference from alternatives with risk of two possible outcomes. Regarding the disadvantages, these methods prove more complex. The ELRO (Equally Likely but Risky Outcomes) and “trade-off”methods are two variants of the equivalent risk methods. The application of ELRO is characterized for:

1.º - To consider two results x_O and x_H, being that $x_H > x_{O;}$

2.º - To consider a reference interval in which the results x_{K1} and x_{K2} are located relatively close to the middle range of the x_O to x_H, where $x_{K2} > x_{K1}$ and $x_{K2} - x_{K1} \sim 1/10\ (x_H - x_O)$;

3.º - To compare the alternatives (x_{k1}, p; x, 1- p) and (x_{k2}, p; x_O, 1-p). Values x_{k1}, x_{k2} and x_O are fixed and the analyst varies x_1 until the decision maker reveals indifference between the two (Coelho, 2005).

According to Coelho (2005), replacing the values in the utility function u it is obtained the following equality for the first indifference:

$$p.u(x_{k1}) + (1\text{-}p).u(x_1) = p.u(x_{k2}) + (1\text{-}p).u(x_O) \quad (3.3)$$

Being p = 0.5, on the ocasion of this substitution we obtain $u(x_1) - u(x_0) = u(x_{k2}) - u(x_{k1})$;

4.º - To compare the alternatives (x_{k1}, p; x_2, 1-p) and (x_{k2}, p; x_1, 1-p). The results x_{k1}, x_{k2} and x_1 are fixed by varying x_2 until the indifference between the two is obtained (Coelho, 2005). The various alternatives presented to the decision maker are obtained, leaving of the determined values as follows: (x_0, p; x_3, 1-p) and (x_1, p; x_2, 1-p), with x_0, x_1 and x_2 fixed, (x_1, p; x_4, 1-p) and (x_2, p; x_3, 1-p), with x_1, x_2 e x_3 fixed, etc. According to Coelho (2005) the sequence is extended until the monetary value is greather than ore equal to x_H.

ELRO is characterized by the need to know the probabilities for, from these, to obtain the utility function and does not have the disadvantages of the certainty equivalent method.

The "trade-off" method was created in the 90´s and more specifically in the year 1996 by Wakker and Deneffe. The main advantage of this method is that the importance of probabilities be reduced and also the fact that preserve the validity of the criterion of expected utility. In the "trade-off" method are considered:

- The p probabilities,
- The reference results x_R and x_r ($x_R > x_r$),
- The minimum result x_0.

Thus:

1.º - It is asked to the decision maker which result x, which makes them indifferent between the (x_1, p; x_r, 1-p) alternative and the (x_0, p; x_R, 1-p) alternative (Coelho, 2005);

2.º - p, x_r, x_0 and x_R are fixed while the analyst does vary x_1 until the decision maker reveals indifference between the two alternatives;

3.º - According to Coelho (2005), then the decision maker´s ask that the result $\mathbf{x_2}$ which makes it indifferent between the pair of (x_2, p; x_r, 1-p) and (x_1, p; x_R, 1-p) alternatives. Again p, x_r, x_1 and x_R remain fixed, varying the x_2 until manifest indifference, between the two of the decision maker.

Substituting the values found in the utility function U gives the following equality for the first indifference (Coelho, 2005):

$$p.u(x_1) + (1-p).u(x_r) = p.u(x_0) + (1-p).u(x_R) \quad (3.4)$$

Thus:

$$p(u(x_1) - u(x_0)) = (1-p).(u(x_R) - u(x_r)) \quad (3.5)$$

We get the equality:

$$p(u(x_2) - u(x_1)) = (1-p).(u(x_R) - u(x_r)) \quad (3.6)$$

Conducting equality of (3.5) and (3.6) and considering $u(x_0) = 0$ it is possible to obtain:

$$u(x_2) = 2\ u(x_1) \quad (3.7)$$

According to Coelho (2005) this procedure continues until a sufficient quantity of results is considered. In this method it is possible to include validation questions, requesting to the decision maker that sets x_K sum, that returns it indifferent between:

$(x_1, p; x_K, 1\text{-}p)$ and $(x_0, p; x_K, 1\text{-}p)$ with $x_K \neq x_r$ and $x_K > x_R$

The same source also notes that can be used x_k and x_K as new reference results instead of x_r and x_R, and verify that the indifference between $(x_{j+1}, p; x_K, 1\text{-}p)$ and $(x_j, p; x_K, 1\text{-}p)$ remains the same for all j.

The gamble-trade-off method, or trade-off method draws inferences from indifferences between two-outcome gambles (as is also done in the lottery-equivalent method) (Wakker and Deneffe, 1996). The method in question (equivalent risk method) is not dependent on the probabilities to calculate the utility and is an obligation the fact that p can not be changed in various alternatives and may be influenced by the probabilities.

As an inconvenient of the method considered is noted that the difference between x_i e x_{i-1} may be identical for all i which it can, according to Coelho (2005) lead to the extraction of behaviours next to neutrality to the risk. In addition, the “trade-off” method requires more work since to extract (n-1) utility values, need to get n indifference - compared with the other methods mentioned that just need a less for observation to extract the same amount of utilities. Another disadvantage that remains in the equivalent risk methods is that the values previously extracted are used in the elicitation of new values causing errors to propagate throughout the process. A disadvantage of this method is that is more laborious when compare with the others methods. For example to elicit n-1 utility values, n indifferences must be observed, while in the CE and PE methods need one observation less to elicit the same utilities. According to Wakker and Deneffe

(1996) this happens because these methods assume one additional parameter given: the probability. Another disadvantages of the TO method:

- responses are "chained": previously elicited values must be invoked to elicit new utility values (Wakker and Deneffe, 1996);
- this method is insensitive to loss aversion and probability transformation.

The major advantage of the trade-off method is its robustness against probability distortions and misconceptions, which constitute a major cause of violations of expected utility and generate inconsistencies in utility elicitation (Wakker and Deneffe, 1996). According to Wakker and Deneffe (1996), the trade-off method retains full validity under prospect theory, rank-dependent utility, and the combination of the two, i.e., cumulative prospect theory. An advantage of the TO method is that gambles with specified probabilities need to be invoked - such gambles may feel artificial and are problematic for subjects who are not familiar with probabilities (Wakker and Deneffe,1996).

3.5 - Others

Wakker and Deneffe (1996), suggest another method: the lottery-equivalent method. This method was introduced, in 1986, by McCord and DeNeufville.

More recently, Villas´is *et al* (2006) propose a modification of existing elicitation utility´s methods: the equal tails (IT). This is a simple method that mitigates the distorting effect of subjective probability weighting. According to Villas´is *et al* (2006) the equal-tails modification of the CE and PE methods, called CE_{ET} and PE_{ET}, respectively - this method constitutes an alternative to the TO to elicit the utility function. The same modification can be applied to the PE elicitation method: PE_{ET}.

4 - Methods´s applications

The various papers which formed the basis for the preparation of this study indicate that the utility elicitation´s methods are usually addressed in disparate sectors, particularly in agriculture and medicine.

4.1 - Agriculture

Table 4.1 is intended as a summary of the methods used to the elicitation of utility, over time, by various investigators and different populations related to the agricultural sector. Of that table are also part of its conclusions.

Table 4.1 - Summary of elicitation utility´s methods used in agricultures and major conclusions (adapt. Coelho, 2005).

Year	*Author/ Investigator*	*Population studied*	*Elicitation utility´method*	*Conclusions*
1968	Officer and Halter	5 australian farmers	Certainty equivalent method	Farmers may be inconsistent, and the utility analysis allows the detection of these inconsistencies.
1974	Lin, Dean and Moore	6 california agricultural producers	ELRO method	The **study´s** results are consistent with the Theory of Expected Utility.
1978	Dillon and Scandizzo	Two samples of small farmers in Northeast Brazil	Certainty equivalent method	The **study´s** results are consistent with the Theory of Expected Utility.
1980	Binswanger	240 farmers in Índia	- Certainty equivalent method; - Experimental game characterized by actual payments.	- When there is an increase in wealth, decreases the risk aversion; - The difference, in the behavior of farmers facing similar technologies and risks, are explained by differences in constraints, such as access to credit.
1982	Herath, Hardaker and Anderson	Rice farmers in Sri Lanka	Certainty equivalent method	The criterion that best predict the behavior of farmers is to maximize the utility with a single attribute.
1996	Bouzit and Gleyses	16 french farmers	- “Trade-off” method; - Certainty equivalent method (function to extract the balance of probabilities).	The results show that the hypothesis of expected utility can be rejected for most farmers.
2003	Pennings and Smidts	332 dutch pig producers	Certainty equivalent method	Local measures of risk aversion may be important to explaining behaviours of operational decision. The structural behavior of decision are related to the overall slope of utility´s curve.

4.2 - Medicine

Health is an important area of applied decision analysis and risky methods are widely used in medical decision analysis (Bleichrodt, Han *et* al, 2005). According to Wakker and Stiggelbout (1993), the standard-gamble (SG) method has been accepted as the “gold standard” for the elicitation of utility when risk or uncertainty is involved in decisions, and thus for the measurement of utility in medical decisions. Unfortunately, there is now abundant evidence that expected utility is not empirically valid, and that the SG method overestimates risk aversion and the utilities of impaired health states (Wakker and Stiggelbout, 1993). You could say that the risk is an important aspect in medical decision making, as one usually faces uncertainty in outcomes of a medical decision. Wakker and Stiggelbout (1993) say that three most common methods for utility elicitation in medical decision making are the time-tradeoff (TTO) method, the visual-analog scale, and the standard gamble (SG) - the first two do not explicity involve risk; hence they lack validity for applications in which uncertainty about outcomes is relevant. Also the same authors say in medical decision making, the SG method conventionally adopts the variable-probability-equivalent method: the probability in the gamble is varied until indifference is obtained between the gamble and the certain outcome - alternatively, one can use a variable-certainty equivalent method, where an indifference between a certain outcome and a risky gamble is obtained by varying the certain outcome.

In the health domain, Stalmeier and Bezembinder (1999), compared riskless strength-of-preference judgments to risky certainty-equivalent judgments for outcomes related with health.

According to Jansen *et al* (_),´s study is necessary to know the effects of treatments (like radiotherapy and chemotherapy) on the life expectancies of cancer patients, but also their effects on the patient´s quality of life. In this case the patient´s preferences are important to the process of decision making on an individual and collective levels. In medicine, the measurement instruments most commonly applied are the visual analog scale (VAS), the time trade-off (TTO), and the standard gamble (SG). The TTO an the SG in their conventional forms measure utilities for chronic health states preferred to death - they are not suitable for the measurement of temporary health states (Jansen *et al*, (_)). VAS is characterized by asking an individual to indicate on a line ranging from 0 (death) to 1 (perfect health) where his/ her preference for a particular health state falls. This method is efficient and easy to use**,** however it does not directly provide cardinal utility values like the SG and the TTO.

5 - Conclusions

The utility measures the satisfaction of the decision maker. Where suggested several elicitation utility´s methods (since 1967 until 2006).

Fishburn (1967) stated twenty-four methods to estimating additive utilities.

In 1982, Hershey *et al* stated the existence of four different methods for constructing von Neumann-Morgenstern utility functions.

Later, in 1984, Farquhar with regard to the "utility assessment methods" consider four new categories.

In the year 2005 Coelho has concluded about the existence of four methods for direct extraction of preferences.

The standard gamble (SG) method and the time tradeoff (TTO) method are commonly used to measure utilities (Osch *et al* (2004).

Hershey and Schoemaker (1985) showed that PE measurements lead to systematically higher utilities than CE measurements (Bleichrodt, Han *et* al, 2005). According the same source, McCord and de Neufville (1986) found that the utility function that is elicited by the CE method depends on the value at which probability is kept fixed.

Trade-off method is less affected by deviations from EU than the CE and PE methods (Wakker and Deneffe, 1996).

Whith respect, in the health domain, of elicitation utility´s methods, several authors have shown different procedures to elicitation of utilities by the PE method that according to expected utility are equivalent lead to systematically different results.

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