Testing serial dependence in the stock markets of the G7 countries, Portugal, Spain and Greece

Paulo Ferreira

CEFAGE-UE
Abstract

This paper utilizes several tests to analyze serial dependence in financial data. In an attempt to provide a better explanation of the behavior of financial markets, we utilized tests that make use of mutual information and developed a detrended fluctuation analysis (DFA). Applying these tests to the series of stock market indexes of 10 European countries, we concluded for the absence of linear autocorrelation. However, with other tests, we found nonlinear serial dependence that affects the rates of return. Our results of mutual information and global correlation based tests confirmed such results. With DFA, we found out that most return rates series have long-range dependence, which appears to be more pronounced for Spain, Greece and Portugal. These conclusions could imply possibility of prediction in those series and thus the violation of the efficient market assumption.

Keywords: serial dependence, stock indexes, mutual information, detrended fluctuation analysis, nonlinearities

JEL Classification: G14, G15
1. Introduction and review of literature

The study of time series’ dependence is one area of interest in economics and management. It is important, for example, in the analysis of financial markets, as the existence of dependency, whether temporal or sectional, could lead to any prediction of the series and the possibility of violating the assumption of efficient markets.

A financial market is considered efficient in its weak form if it is not possible to identify any deterministic pattern in its time series’ behavior. This means that there is no possibility, through arbitrage, of obtaining systematic abnormal profits using past information (Fama, 1970). In other words, return rates have no memory. For this reason, financial markets have been subject to extensive analysis to check whether there are windows of profit opportunities, considering the fluctuations and dynamics of markets themselves (see, for example, the review of Pagan, 1990).

One of the first studies applied to the behavior of financial markets’ series was developed by Bachelier (1900), who studied the probability distribution of share prices and concluded that prices follow a Gaussian distribution. Kendall (1953) found out that stock prices are randomly determined. The author even made an analogy with the results obtained in a roulette, which implies that return rates are independent and identically distributed (iid). Some other studies, namely those by Osborne (1964), Working (1960), Moore (1964), Granger and Morgenstein (1964) or Fama (1963) validated the random walk hypothesis, which means that asset prices have no memory and are therefore independent in time. For a long time Bachelier’s theory (1900) that financial series behave like a random walk was accepted and introduced in many economic models, such as the efficient markets hypothesis proposed by Fama (1970).

However, several studies contradicted this evidence, finding the existence of stylized facts (see, for example, Cont 2001). One of these stylized facts is the existence of fat tails in returns distributions, which is related to the fact that the volatility of assets returns is higher than expected by a Gaussian distribution. This implies a leptokurtic curve, which means that return rates are not independent and identically
distributed (see, eg, Mandelbrot 1964, Osborne, 1964 or Mantegna et al. 1999.). Cont (2001) indicated that, although it is difficult to accurately determine the behavior of distributions tails, their existence lead to the exclusion of stability and thus to the rejection of the fact that return rates follow a normal distribution. However, the author concluded that results depend on the time scale of the analysis and that for bigger time scales there is an approximation to a normal distribution. On the other hand, the study showed that there is no linear correlation between return rates, except for relatively small periods of time (less than 20 minutes) related to the microeconomic structure of certain events. However, in absolute terms, in some cases autocorrelation functions decay slowly, which may indicate the existence of time dependence. Cont (2001) also identified other stylized facts that may contribute to reject the evidence of normality in assets returns, including the existence of asymmetries in gains and losses (loss movements are more pronounced); greater than the expected intermittency and variability of returns, with volatility clustering behavior (events with high volatility tend to cluster in time); leverage effect (negative relation between volatility and profitability); correlation between trading volumes and volatility; and existence of autocorrelation in variance.

The existence of nonlinearities in time series can be understood as a failure of efficient market hypothesis in its weak form. Under this hypothesis, participants in financial markets cannot make systematic profits as markets internalize the information. In the absence of linear autocorrelation we could not conclude on the existence of efficient financial markets, since there may be other nonlinear relations that affect the behavior of a financial asset. We must be also aware that even the verification of some kind of dependence may not necessarily mean inefficiency of markets since the existence of transaction costs can nullify the possibility of making arbitrage transactions (see, for example, Fama, 1970).

These facts have been a frequent object of discussion in financial theory and many researchers have assessed the existence of dependence in financial markets. This dependence can be analyzed in a time series (over time) or between series. Providing different information, both are interesting to analysts: while
time dependence can affect the predictability of a particular series, dependence between series may answer some interesting questions, including the possibility of contagion effects.

In fact, analyses of serial dependence, both linear and nonlinear, had some relevance in the financial literature in recent times. In most cases, empirical studies identified the possibility of autocorrelation. However, generally these linear autocorrelations quickly disappear, although there are authors defending the existence of long-term dependence (see, e.g., Campbell 1987). Much of the interest in these problems comes from statistical physics’ authors, with an increasingly approximation between this research area and economics. The high amount of data that is available and the complexity of these markets are other features that have attracted these researchers to study financial markets (see, for example, the review of Chakraborti et. al, 2011).

Relevant studies on market efficiency used linear equations to analyze return rate dependence, failing to detect other types of dependency, including non-linear dependence. This means that, even not rejecting the hypothesis that returns have no autocorrelation, it is possible that markets are not efficient if there is any other kind of dependence and therefore analysis of linear dependence is not sufficient (see, for example, Darbellay 1998a, Maasoumi and Racine, 2002 and Granger et al. 2004). Therefore, in order to study financial markets, it is necessary to follow general models which are capable of capturing global, and not only linear, dependence.

In this context, mutual information was introduced and its properties were explored as a measure of dependence in time series. This method has some advantages, because it considers the whole structure of time series, linear and nonlinear (see, for example, Darbellay and Wuertz, 2000).

Later, a test that uses mutual information in order to analyze statistical dependence of time series was created, with critical values provided by Dionísio et al. (2006). The use of mutual information follows the conclusions of several authors who argued that measures relating to the information theory are very

Recently, new methods to analyze long-term dependence in time series have been developed. One of these methods is detrended fluctuation analysis (DFA). DFA was created by Peng et al. (1994) and studies the behavior of individual series. It has been used in various fields of analysis, such as health, cardiology or weather. Several studies were also developed in the fields of economics, with particular emphasis on financial markets. DFA has the advantage of allowing conclusions of non-stationary series, which are common in some financial series. In this paper we use DFA to analyze the behavior of stock markets. Later on, with the presentation of this methodology, there is a more detailed review of literature for studies that use this methodology.

This paper analyzes the behavior of stock indices for the G7 (Germany, USA, UK, Italy, Japan, France and Canada) and also for Portugal, Spain and Greece. Initially we analyze the behavior of the series of returns through the usual descriptive statistics. This analysis is intended to check the existence of some sort of linear relationship (it is possible to do so using autocorrelation tests). This preliminary assessment is followed by the analysis of non-linear dependence with tests of nonlinearity complemented by the use of mutual information, which have some advantages.

The remaining of the paper is organized as follows: Section 2 presents some tests and methodologies that are applied in this paper, Section 3 reports the empirical analysis and its results and Section 4 concludes.

2. Methods to analyze dependence in financial time series

One critical issue in financial economics is the assessment of time dependence, which is related to the efficient markets hypothesis. There are several tests to analyze time dependence in financial series and
some are used in this paper: BDS test (Brock et al., 1996), Engle test to analyze ARCH effects (Engle, 1982), McLeod and Li test (McLeod and Li, 1983) and Tsay test (Tsay, 1986). These tests are quite well known in the financial literature and are therefore not described here. Besides these tests, we also analyze dependence with mutual information and with DFA, which are addressed below.

2.1. Mutual information

Mutual information gives the common information between two (or more) different distributions. Introduced in the literature by Shannon (1948), this concept has been improved and widely used over time. In the context of time series, it is used to analyze dependence over time. Mutual information can be understood as a measure of dependence or correlation. However, we should be careful in its interpretation, as it does not provide indication of causality between variables. Mutual information is given by the following expression:

\[
I(X,Y) = \int \int p_{X,Y}(x,y) \log \left( \frac{p_{X,Y}(x,y)}{p_X(x)p_Y(y)} \right)
\]

(1)

It can take any positive value or may be zero. It will be zero if variables are independent (and therefore have no information in common). According to Granger et al. (2004), this makes mutual information an imperfect measure of dependence, since it does not take absolute values between 0 and 1 only. It is therefore necessary to standardize it to make direct comparisons (see, for example, Granger and Lin, 1994, Darbellay, 1998b and Soofi, 1997). One possible normalization is:

\[
\hat{I}(X,Y) = \sqrt{1 - e^{-2I(X,Y)}}
\]

(2)

The measure of dependency identified by equation (2) could vary between 0 and 1 and can be interpreted as a correlation that is based on information theory, taking the 0 value if the variables X and Y are independent (i.e. if the variables do not have information in common). The maximum value is obtained in the case of a perfect relationship between two variables, i.e., in a deterministic context.
It is used as an alternative to other tests because it presents several advantages. Firstly, some of the previous tests have some limitations. For example, the Pearson correlation coefficient only captures the existence of linear correlations, but non-linear correlations may also be present in the data. Thus, mutual information may be used as a measure of overall correlation and not just of linear correlation. For this reason it is irrelevant if the sign of the relationship is positive or negative. Moreover, measures related to entropy require fewer assumptions and are more flexible.

Mutual information is used to test global dependence of a time series. The null hypothesis is defined as $H_0$: $I(X,Y) = 0$, meaning that variables are independent (or that a given time series has no memory). The alternative hypothesis is given by $H_1$: $I(X,Y) > 0$. The decision of rejecting or not rejecting the null hypothesis is made by comparing the relevant values with the critical ones calculated by Dionísio et al. (2006). This test has the particularity of not needing assumptions on the linearity, normality or stationarity of time series. However results are more robust in the case of stationary time series because there is insufficient evidence of the robustness of this test when nonlinearity and nonstationarity simultaneously occur (see Fernandes, 2001).

We estimate mutual information by the equiquantization method\(^1\).

### 2.2. Detrended Fluctuation Analysis

Exploring the possibility of chaos in financial markets has been a recurrent topic of analysis in several studies. If a time series is described by a random walk, then there is no verification of chaos in it. The Hurst exponent is a statistic used to distinguish between random or not random behavior of a time series. Initially used by Harold Hurst in determining the randomness in the behavior of the Nile River (see, for example, Crato 1994), it was generalized to other natural phenomena which display a non random (noise) behavior. In addition, this type of analysis has also been used in economics, particularly in financial economics (see, \(^2\)

\(^1\) For some discussion about the choice of the methods to estimate mutual information, Moddemeijer (1989 and 1999), Darbellay (1998a), Bernhard et al. (1999, Kraskov et al. (2004) or Granger et al. (2004).
for example, Peters, 1996). Basically, a Hurst exponent different from 0.5 contradicts the hypothesis of randomness and hence of efficient markets.

Although this paper does not directly use the Hurst exponent, we apply a methodology that indirectly provides the same information: DFA, a technique used to analyze temporal dependence in time series with the advantage of being used in the context of nonstationary time series.

This methodology was developed by Peng et. al (1994), originally to study the behavior of DNA, but since then it has been used to analyze many different problems, from heartbeats to the behavior of financial series. The main objective of this technique is to analyze the relationship between values $x_t$ and $x_{t+s}$ at different moments in time. The different steps to apply DFA could be found in Peng et. al (1994).

The objective of this method is to model the behavior of time series as a power-law. The $\alpha$ parameter is equivalent to the Hurst exponent, used to analyze serial dependence. If $\alpha = 1/2$, this means that time series is represented by a random walk, so the autocorrelation function is zero for any period of time and the process do not have a long memory. If $\alpha \neq 1/2$, it implies the existence of long-term correlations in the considered time interval. There is a positive long-range dependence (the series are persistent), in the case of $1/2 < \alpha < 1$. If, instead, the value is positive but $\alpha < 1/2$, this indicates a negative long-range dependence (anti-persistence) meaning that larger fluctuations are followed by smaller fluctuations (or vice versa). If the value of $\alpha$ is equal to 1, the process is a pink noise. If it is greater than 1, it shows the existence of long-term dependence but cannot be analyzed according to a power-law. You can graphically analyze the behavior of this parameter on log-log graph for values of $\langle F(s) \rangle$ and for the time scale.

Some authors applied this technique to financial markets. Cizeau et al. (1997) concluded that correlations of returns on S&P500 quickly vanish, but that their absolute values did not show this effect, which shows nonlinearity in returns’ volatility. In turn, Ausloos et al. (1999) analyze foreign exchange markets, comparing the behavior of exchange rates of the German mark and the Polish zloty, both against the Belgian franc, concluding in favor of temporal dependence in these series. Ausloos (2000) also analyzed
foreign exchange markets, studying 13 different exchange rates and found out that, in most cases, there was evidence of long-term correlations. Jaroszewicz et. al. (2005) developed a pioneer work, because they analyzed Latin American indexes. In their study, the authors found evidence of correlations of long-term returns primarily in absolute returns and in relation to return rates. The behavior of the assessed series indicated a slow approximation to the Gaussian distribution. However, as sample size was relatively small, the authors had some caution with their conclusions. Alvarez-Ramirez et al. (2008) applied DFA to evaluate the possibility of forecasting ability in oil prices, concluding that, in short periods of time, there was some persistence in the correlations, but for longer horizons (time spans greater than 25 days) such relationship ceased to exist. Analyzing 26 different stocks of the NYSE on a given day, Mariani et. al (2009) concluded that 19 out of 26 titles presented evidence of long memory, of which 18 had persistent behavior and only one was anti-persistent. Muchnik et al. (2009) did not use DFA to analyze returns or volatility directly, but rather to analyze the sequence of maxima and minima in the behavior of various assets (stocks’ prices and foreign exchange rates). The authors concluded that a long-term correlation existed between these maximum and minimum values, relating such results to volatility clustering.

3. Results of serial dependence in the stock markets

3.1 Preliminary results

In this study we use daily data, from stock market indexes for 10 different countries: G7 group (Germany, Canada, United States, France, Italy, Japan and the United Kingdom) as well as Portugal, Spain and Greece, three Union European countries with some similar characteristics. Time series consist of 5,356 observations, with data from 2 January 1990 to July 13, 2010 and were collected from the DataStream database (five days week, daily closing prices and from Datastream stock indexes). Figure 1 shows the evolution of those indexes.
Figure 1. Evolution of stock indexes between 02/01/1990 and 13/07/2010

A brief visual analysis suggests that, despite the distinct fluctuation patterns, the majority of the indexes display two periods of pronounced maxima: one around 2000 and the other around 2007. There is however an index with a different behavior: that of Japan. Despite having the same two peaks, it seems to have a more volatile behavior when compared with the remaining indexes, indicating a decrease in the early sample period, which does not happen in other indexes. In fact, this is the only index that shows a decline in stock market value during this period, with a loss of 66%. All the other indexes display an overall growth pattern, with Italy growing 42% in the period and Greece having the highest growth rate (370%).

Financial time series analysis requires the assessment of stationarity to help decide on the most appropriate methods to be used. Traditionally, the Dickey-Fuller test (or its expanded version) is one of the most widely used. However this test should be used with caution in case there are structural breaks in the series. In such case, the test’s results can be misleading. Therefore, the test of Perron and Vogelsang (1992)
is used in this study to assess the stationarity of a series, while checking whether there are structural
breaks. According to Perron (1997), these tests are preferable because if one can reject the hypothesis of a
unit root in a time series with a structural break, one can also reject it with weaker assumptions.

These tests were applied to the indexes of stock prices. Except for one of the cases (Greece), there seems
to be breaks in all series. In addition, and excepting the case of Japan where the series are stationary, all
other cases are nonstationary and are integrated of order one\(^2\).

A statistical analysis of the series of rates of return is now developed, in order to assess their characteristics
and to try to identify possible sources of stylized facts. The daily returns are calculated as usual:

\[
R_t = \ln P_t - \ln P_{t-1}
\]

where \(R_t\) is the return rate of day \(t\), \(P_t\) and \(P_{t-1}\) are close quotations for different indexes.

Figure 2 shows the evolution of these rates of return in the considered sample period. Although no strong
conclusions can be drawn from simple graphical analysis, it seems that the Greek index is the one displaying
the higher volatility.

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\(^2\) Due to space constraints, results are not shown here, but are available upon request.
We now calculate some important statistics to analyze the indexes in study. Firstly, Japan is the only index displaying a negative mean in return rates. It is also confirmed what graphic analysis appeared to indicate: Greek index is the one with higher volatility, measured by standard deviation.

We also study the normality of return rates. Through Jarque-Bera test it is possible to conclude that we always reject null hypothesis of normality and also that the kurtosis of distributions is always very high when compared to a Gaussian distribution. Such results may be explained by the existence of fat tails.

3.2 Dependence, linearity and mutual information

We assess the behavior of the rates of return by testing the existence of autocorrelation (with a Breusch-Godfrey’s test), of heteroskedasticity (with a White’s test) and of autoregressive heteroskedasticity (with an Engle’s test).
In the autocorrelation tests we never reject the null hypothesis, which means that linear autocorrelation in stock returns apparently do not exist. However, as mentioned earlier, this does not necessarily imply the absence of temporal independence, since nonlinearities can be present in the series. Heteroskedasticity or ARCH effects are some of possible nonlinearities to be detected in financial series. In fact, for all countries, we reject the null hypothesis for these tests, which is evidence supporting the existence of heteroskedasticity and autoregressive heteroskedasticity, and indicating the possibility of nonlinearities in stock returns. These results are presented in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>USA</th>
<th>UK</th>
<th>Ger</th>
<th>Jap</th>
<th>Can</th>
<th>Fra</th>
<th>Ita</th>
<th>Gre</th>
<th>Spa</th>
<th>Por</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AR model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ARCH - F</strong></td>
<td>159,56**</td>
<td>193,06**</td>
<td>73,27**</td>
<td>145,78**</td>
<td>210,19**</td>
<td>137,57**</td>
<td>120,27**</td>
<td>67,57**</td>
<td>57,63**</td>
<td>138,27**</td>
</tr>
<tr>
<td><strong>lags</strong></td>
<td>12</td>
<td>7</td>
<td>13</td>
<td>10</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td><strong>BG - F</strong></td>
<td>0,008</td>
<td>1,62</td>
<td>2,87</td>
<td>1,92</td>
<td>0</td>
<td>2,23</td>
<td>1,7</td>
<td>1,84</td>
<td>2,54</td>
<td>0,59</td>
</tr>
<tr>
<td><strong>lags</strong></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>White</strong></td>
<td>222,37**</td>
<td>78,94**</td>
<td>147,96**</td>
<td>218,76**</td>
<td>96,36**</td>
<td>75,66**</td>
<td>144,77**</td>
<td>56,74**</td>
<td>141,98**</td>
<td>185,08**</td>
</tr>
</tbody>
</table>

* denote significance at 1% level and ** denote significance at 5% level

The McLeod and Li test is an alternative to analyze conditional heteroskedasticity. As it appears that there is no evidence of autocorrelation, there are no problems with the use of this test. We always reject the null hypothesis which also points for the existence of dependence in time series. Both Engle and McLeod and Li tests provide evidence of nonlinearity in variance. The occurrence of this phenomenon can be related, according to Scalas (2005), to the existence of volatility clusters. Results are presented in Table 2.

The study proceeds with the BDS test, used to check for independence of time series. We always reject the null hypothesis of independence, so the series show evidence of time dependence. The results are presented in Table 3.
The Tsay test also allows us to analyze nonlinearity but in the mean of a time series and not in its variance.

In this case we can conclude that, with the exception of Germany and Spain, all the other series show nonlinearities for its mean. The Tsay test’s results are presented in Table 4.

**Table 2.** Mcleod and Li test applied to return series where \( k \) represents respective lag.

<table>
<thead>
<tr>
<th>( k )</th>
<th>Ger</th>
<th>Can</th>
<th>Spa</th>
<th>USA</th>
<th>Fra</th>
<th>Gre</th>
<th>Ita</th>
<th>Jap</th>
<th>Por</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.211**</td>
<td>0.305**</td>
<td>0.188**</td>
<td>0.209**</td>
<td>0.202**</td>
<td>0.203**</td>
<td>0.193**</td>
<td>0.155**</td>
<td>0.213**</td>
<td>0.225**</td>
</tr>
<tr>
<td>2</td>
<td>0.137**</td>
<td>0.218**</td>
<td>0.181**</td>
<td>0.355**</td>
<td>0.247**</td>
<td>0.157**</td>
<td>0.231**</td>
<td>0.377**</td>
<td>0.174**</td>
<td>0.284**</td>
</tr>
<tr>
<td>3</td>
<td>0.127**</td>
<td>0.256**</td>
<td>0.174**</td>
<td>0.198**</td>
<td>0.234**</td>
<td>0.170**</td>
<td>0.230**</td>
<td>0.183**</td>
<td>0.165**</td>
<td>0.310**</td>
</tr>
</tbody>
</table>

** denote significance at 1% level and * denote significance at 5% level

**Table 3.** BDS test applied to return series where \( k \) represents dive dimension

<table>
<thead>
<tr>
<th>( k )</th>
<th>Ger</th>
<th>Can</th>
<th>Spa</th>
<th>USA</th>
<th>Fra</th>
<th>Gre</th>
<th>Ita</th>
<th>Jap</th>
<th>Por</th>
<th>UK</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>0.012**</td>
<td>0.014**</td>
<td>0.009**</td>
<td>0.009**</td>
<td>0.007**</td>
<td>0.015**</td>
<td>0.009**</td>
<td>0.006**</td>
<td>0.022**</td>
<td>0.010**</td>
</tr>
<tr>
<td>3</td>
<td>0.014**</td>
<td>0.016**</td>
<td>0.010**</td>
<td>0.012**</td>
<td>0.008**</td>
<td>0.017**</td>
<td>0.011**</td>
<td>0.007**</td>
<td>0.026**</td>
<td>0.011**</td>
</tr>
<tr>
<td>4</td>
<td>0.011**</td>
<td>0.013**</td>
<td>0.008**</td>
<td>0.010**</td>
<td>0.006**</td>
<td>0.014**</td>
<td>0.008**</td>
<td>0.005**</td>
<td>0.022**</td>
<td>0.008**</td>
</tr>
<tr>
<td>5</td>
<td>0.007**</td>
<td>0.009**</td>
<td>0.005**</td>
<td>0.007**</td>
<td>0.003**</td>
<td>0.009**</td>
<td>0.005**</td>
<td>0.003**</td>
<td>0.016**</td>
<td>0.005**</td>
</tr>
</tbody>
</table>

Standard deviation equal to 0.5 ** denote significance at 1% level and * denote significance at 5% level

**Table 4.** Tsay test applied to return series

<table>
<thead>
<tr>
<th>Country</th>
<th>F Statistic</th>
<th>d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ger</td>
<td>2,7411</td>
<td>(1, 5351)</td>
</tr>
<tr>
<td>Can</td>
<td>15,2882**</td>
<td>(21, 5411)</td>
</tr>
<tr>
<td>Spa</td>
<td>2,0342</td>
<td>(1, 5351)</td>
</tr>
<tr>
<td>USA</td>
<td>8,9186**</td>
<td>(3, 5347)</td>
</tr>
<tr>
<td>Fra</td>
<td>6,2352**</td>
<td>(15, 5329)</td>
</tr>
<tr>
<td>Gre</td>
<td>7,7641**</td>
<td>(6, 5342)</td>
</tr>
<tr>
<td>Ita</td>
<td>8,5055**</td>
<td>(1, 5351)</td>
</tr>
<tr>
<td>Jap</td>
<td>3,5128**</td>
<td>(3, 5347)</td>
</tr>
<tr>
<td>Por</td>
<td>13,4648**</td>
<td>(1, 5351)</td>
</tr>
<tr>
<td>UK</td>
<td>5,6296**</td>
<td>(21, 5411)</td>
</tr>
</tbody>
</table>

** denote significance at 1% level and * denote significance at 5% level
As previously stated, mutual information can also be used to test independence in a statistical distribution, being preferable to other measures such as the linear correlation coefficient, since mutual information is a measure of overall correlation between data and not solely a linear correlation. First, we calculate mutual information for daily return rates, considering the first 10 lags. Results point to the existence of a strong dependence in the data. The only exception is the eighth lag for Japan whose value is not statistically significant. Excepting the UK and the north-american markets, mutual information decreases when the number of lags increases.

We also calculate overall correlation coefficients based on mutual information. While linear correlation coefficients are relatively small in most cases, the same does not occur with global correlation coefficients, showing evidence that there are probably nonlinearities that must be considered in the behavior of the rates of returns. Figure 3 illustrates these differences. Overall, correlation coefficients are represented by lighter gray bars, while linear correlation coefficients are represented by darker bars. To simplify, we use linear correlation coefficients in absolute values.
Figure 3. Global correlation coefficients and linear correlation coefficients for return rate series. Global correlation coefficients are represented by gray bars; linear correlation coefficients are represented by dark bars.

For first lags, the German index is the one with the highest value for mutual information, followed by the Portuguese, the Greek and the Spanish indexes. This may indicate a strong nonlinear dependence on these indexes. Although in general mutual information values decrease as the number of lags increase, this does not happen in all markets (such as in Canada, USA or UK). Still, coefficients on the 10th lag remain significant, so that the memory of the rates of return is not so short. Some studies, such as Bonanno et al.
(2001) and Mantegna et al. (1999), indicate that the autocorrelation function is usually monotonically decreasing over time. Our results support this feature, when we consider its linear component.

If we order the indices by the values of the overall correlation coefficients and by linear correlation coefficients the order is not the same. This may mean that linear and nonlinear components affect each index differently. Orders are not constant, but reinforce the idea that the German index is the one with larger nonlinear correlations, although for linear correlations its values are the lowest. This allows us to confirm our previous conclusion: there is probably a strong nonlinear dependence for this index.

For all indexes, overall correlation coefficients are higher than linear correlation coefficients which, in some cases, are almost null. In addition, mutual information estimations provide results that are statistically significant for all lags, except for one case for Japan, as previously mentioned. These results indicate, therefore, the possibility of nonlinear dependence in the series of returns. However, it must be stressed that the difference between global and linear coefficients cannot be taken as evidence of nonlinear dependence in the series under assessment.

We filtered the return rate series using the residuals of the Engle’s test to try to isolate possible sources of nonlinear dependence. We continue this study with the BDS and the mutual information tests on the filtered series.

BDS test results for the filtered series continue to show evidence of time dependence, even after filtering. The results are presented in Table 5.

Table 5. BDS test applied to filtered return series where $k$ represents dive dimension (standard deviation equal to 0,5).

<table>
<thead>
<tr>
<th>$k$</th>
<th>Ger</th>
<th>Can</th>
<th>Spa</th>
<th>USA</th>
<th>Fra</th>
<th>Gre</th>
<th>Ita</th>
<th>Jap</th>
<th>Por</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0,012**</td>
<td>0,015**</td>
<td>0,009**</td>
<td>0,009**</td>
<td>0,007**</td>
<td>0,016**</td>
<td>0,009**</td>
<td>0,006**</td>
<td>0,021**</td>
<td>0,009**</td>
</tr>
<tr>
<td>3</td>
<td>0,014**</td>
<td>0,016**</td>
<td>0,010**</td>
<td>0,012**</td>
<td>0,008**</td>
<td>0,017**</td>
<td>0,011**</td>
<td>0,007**</td>
<td>0,025**</td>
<td>0,010**</td>
</tr>
<tr>
<td>4</td>
<td>0,011**</td>
<td>0,013**</td>
<td>0,008**</td>
<td>0,009**</td>
<td>0,006**</td>
<td>0,013**</td>
<td>0,008**</td>
<td>0,005**</td>
<td>0,021**</td>
<td>0,008**</td>
</tr>
<tr>
<td>5</td>
<td>0,007**</td>
<td>0,009**</td>
<td>0,005**</td>
<td>0,007**</td>
<td>0,003**</td>
<td>0,009**</td>
<td>0,005**</td>
<td>0,003**</td>
<td>0,016**</td>
<td>0,005**</td>
</tr>
</tbody>
</table>
We also calculate mutual information and global correlation coefficients for the filtered series. Once again, the results point out the existence of dependence as the null hypothesis is always rejected, excepting one lag for Japan, with 1% of significance. In addition, the Japanese index shows two values that are only significant at 5% level. In terms of comparison of mutual information coefficients, most of the values are lower for the filtered series. We can also see that the global correlation coefficients are much higher than linear correlation coefficients, which suggests the possibility of nonlinear dependence, also in the filtered series (Figure 4).
**Figure 4.** Global correlation coefficient and linear correlation coefficient for filtered return rate series.

Global correlation coefficients are represented by grey bars; linear correlation coefficients are represented by dark bars.

Comparing Figures 3 and 4 suggests that the filtering process eventually has an effect in reducing the correlation coefficients, as expected. For linear correlation coefficients, reductions are very pronounced. For example, for the first lag, there is almost no linear correlation in the filtered series. Indexes such as the Portuguese and the Greek, which have relatively high global correlation values, with the filtering process have correlation coefficients that are practically zero. This means that the filtering process significantly reduces linear correlations.
In most cases, the filtering process also reduced the global coefficients. However, such reduction is far less significant than in the previous case. It is therefore possible to conclude that, eventually, some types of nonlinear correlation remain in the stock indexes returns, even after the filtering process. Not being able to identify the type of nonlinearities present in the data, an analysis with mutual information and with global correlation coefficients allows the identification of the lags presenting greater evidence of nonlinear dependence. The results indicate the possibility of this type of dependence, both in the series of returns and of filtered returns. This could mean that the series allow some predictability.

3.3. DFA application to return rates

The DFA application is performed using the R software, assuming a linear trend for a total of 65 lengths boxes, from 4 to 2610. This methodology is applied to the series of returns and of filtered return. Recovering information for all box lengths, it is possible to calculate the values for the $\alpha$ parameter but also for its standard deviation, and to test the hypotheses $H_0: \alpha = 0.5$ and $H_1: \alpha \neq 0.5$. For the rates of return, the values of $\alpha$ are all very close of 0.5. However, the Japanese index is the one where we do not reject the null hypothesis, which means that for the others, though not very pronounced, there is a long-term dependence in the returns, with persistent characteristics (since the parameter is greater than 0.5). Countries for which the long-term relationship is most marked are Spain, Greece and Portugal, all countries of the Southern Europe, normally associated with less robust economies and also those who have less liquid stock markets.

The results for the filtered returns are very similar, with the difference that in addition to Japan, the coefficient of Canada is also equal to 0.5. In most cases, as expected, the degree of dependence in filtered returns is lower. However, the fact that results remain above 0.5 means that there are long-term dependencies even after the filtering of data. Results are given in Table 6.
Table 6. Hypotheses tests for DFA results.

<table>
<thead>
<tr>
<th>Country</th>
<th>( \hat{\alpha} )</th>
<th>s.d.(( \hat{\alpha} ))</th>
<th>( \hat{\alpha} )</th>
<th>s.d.(( \hat{\alpha} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>0,5219**</td>
<td>0,0065</td>
<td>0,5148**</td>
<td>0,0058</td>
</tr>
<tr>
<td>UK</td>
<td>0,5102**</td>
<td>0,004</td>
<td>0,5182**</td>
<td>0,0047</td>
</tr>
<tr>
<td>Germany</td>
<td>0,5319**</td>
<td>0,0053</td>
<td>0,5238**</td>
<td>0,0053</td>
</tr>
<tr>
<td>Japan</td>
<td>0,5006</td>
<td>0,0072</td>
<td>0,5035</td>
<td>0,0072</td>
</tr>
<tr>
<td>Canada</td>
<td>0,5397**</td>
<td>0,005</td>
<td>0,5156*</td>
<td>0,0076</td>
</tr>
<tr>
<td>France</td>
<td>0,5258**</td>
<td>0,006</td>
<td>0,5269**</td>
<td>0,00596</td>
</tr>
<tr>
<td>Italy</td>
<td>0,5296**</td>
<td>0,007</td>
<td>0,5237**</td>
<td>0,0069</td>
</tr>
<tr>
<td>Greece</td>
<td>0,5575**</td>
<td>0,0069</td>
<td>0,5389**</td>
<td>0,0062</td>
</tr>
<tr>
<td>Spain</td>
<td>0,5440**</td>
<td>0,0061</td>
<td>0,5218**</td>
<td>0,0057</td>
</tr>
<tr>
<td>Portugal</td>
<td>0,5855**</td>
<td>0,0068</td>
<td>0,5665**</td>
<td>0,0067</td>
</tr>
</tbody>
</table>

\( \hat{\alpha} \) is the estimated parameter and s.e.(\( \hat{\alpha} \)) its standard deviation. ** denote significance at 1% level and * denote significance at 5% level.

Countries where a long-term relationship in rates is more pronounced are Spain (only for the series of returns before filtering), Portugal and Greece (in both cases), since the parameter values estimated by DFA are higher. South European countries are those which have less robust economies, which could influence these results. In addition, they are also countries where stock markets are less developed which could lead to a lower degree of liquidity in these markets, which may also help to explain these results.

It is also relevant that results for the German index show higher persistence, even in the case of the filtered series, with a value that is higher than the Spanish index. As previously mentioned, the fact that this sample includes the reunification period can justify such results. Even during the analysis using the mutual information and the global correlation coefficients, there was evidence of high values for Germany.
4. Conclusions

The behavior of financial series is usually analyzed in order to identify the existence of dependences. Such assessment is important, for example, to evaluate the hypothesis of efficient markets. A financial market is efficient when it is not possible to identify deterministic patterns in its time series, which means that there is no possibility of making systematic profits with arbitrage.

We analyzed rates of return for 10 different stock market indexes (for the G7 countries plus Portugal, Spain and Greece) and applied several tests using data from 1990 to mid-2010, a total of 5,356 daily observations. We began by testing for stationarity. Due to the existence of breaks, robust tests were used and their results suggest that most of series are nonstationary with a break.

In the analysis of the behavior of return rates, we rejected the hypothesis of normality. Our results also point out the possibility of fat tails, which is a stylized fact in the financial literature, and indicates the possibility of nonlinear time dependence (Cont 2001). The autocorrelation tests’ results indicate that the rates of return do not suffer from this problem. However, this is an analysis that will capture the existence of linear relationships only. When we analyzed nonlinear effects, our results indicated the existence of nonlinearities in series of returns. These results were confirmed by other tests to nonlinearity, such as the BDS and the Tsay tests. Also using mutual information and global correlation coefficients, it was suggested that nonlinearities could exist in the rates of return, with the German index being one for which more nonlinear correlations existed. Germany’s reunification could be an explanation for this result.

We continued with the filtering of the series, in an attempt to isolate nonlinear dependence sources. However, the results continued to suggest evidence of time dependence in the stocks’ returns.

The DFA was also applied to the returns, before and after filtering. In what concerns the rates of return, they presented evidence of proximity to a random walk, but the results remain statistically different from this hypothesis, which means that there is some evidence of long-term dependence. Spain, Greece and Portugal are the countries with a more marked long-term dependency. Only the Japanese case presents
returns that have no memory. Applying the method to the series of filtered returns, the obtained results are qualitatively similar, and in addition to the Japanese index, the Canadian has also supported the random walk. Once again, the German index and the indexes of the Southern European countries in our sample presented higher values.

Acknowledgement: This work was supported by Fundação para a Ciência e Tecnologia (FCT) [grant FCOMP-01-0124-FEDER-007350].

References


