An Alternative Reference Scenario for Global CO2Emissions from Fuel Consumption: An ARFIMA Approach

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Abstract

We provide alternative reference forecasts for global CO₂ emissions based on an ARFIMA model estimated with annual data from 1750 to 2013. These forecasts are free from additional assumptions on demographic and economic variables that are commonly used in reference forecasts, as they only rely on the properties of the underlying stochastic process for CO₂ emissions, as well as on all the observed information it incorporates. In this sense, these forecasts are more based on fundamentals. Our reference forecast suggests that in 2030, 2040 and 2050, in the absence of any structural changes of any type, CO₂ would likely be at about 25%, 34% and 39.9% above 2010 emission levels, respectively. These values are clearly below the levels proposed by other reference scenarios available in the literature. This is important, as it suggests that the ongoing policy goals are actually within much closer reach than what is implied by the standard CO₂ reference emission scenarios. Having lower and more realistic reference emissions projections not only gives a truer assessment of the policy efforts that are needed, but also highlights the lower costs involved in mitigation efforts, thereby maximizing the likelihood of more widespread energy and environmental policy efforts.

Keywords: Forecasting, reference scenario, CO₂ emissions, long memory, ARFIMA.
JEL Codes: C22, C53, O13, Q47, Q54.

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1. Introduction

There is a great degree of debate on how to specify long-term reference case scenarios for CO$_2$ emissions. This is critical first step in ascertaining the extent of the policy effort required to achieve any policy target for CO$_2$ emissions, and thereby determining the costs involved in achieving such goals.

In the most fundamental sense, specifying a reference scenario means forecasting a path for CO$_2$ emissions that reflects not only the existing economic and demographic trends, but also the ongoing emissions policies. Most reference case forecasts, however, are based on a multivariate approach and include alternative economic and demographic assumptions, as well as very often even new policy commitments as in the typical “business as usual” projections [see, for example, IEA (2007), OECD (2012), and EIA (2013)]. The problem with this conventional approach to establishing reference scenarios is that it introduces rather a great degree of arbitrariness in their definition, and thereby clouds the information it intends to provide – specifically, the extent of the efforts needed and the corresponding costs.

This note uses an ARFIMA approach to provide reference forecasts for global CO$_2$ emissions based on a univariate statistical analysis, recognizing the presence of long-memory through fractional integration. Our forecasts for CO$_2$ emissions are strictly based on the most basic statistical fundamentals of the stochastic process that underlies global CO$_2$ emissions. As such, our forecasts capture the information included in the sample, and implicitly assume that the observed trends will continue in the future. Thus, these forecasts provide the most fundamental reference case forecast of CO$_2$ emissions.

The methodology we adopt is inspired by a budding literature on the analysis of energy and carbon emissions based on a fractional integration approach [see, for example, Elder and Serletis (2008), Lean and Smyth (2009), Gil-Alana et al. (2010), Barassi et
al. (2011), Apergis and Tsoumas (2011, 2012), Barros et al. (2012a, 2012b), Liu and Chen (2013), and Gil-Alana et al. (2015)]. Measuring the persistence of CO$_2$ emissions is of utmost importance for the design of energy and environmental policies aiming at reducing the economy’s carbon intensity. If CO$_2$ emissions are stationary, then transitory public policies will tend to have only transitory effects. Permanent changes, therefore, require a permanent policy stance. On the other hand, if CO$_2$ emissions are not stationary, then even transitory policies will have permanent effects on emissions, and a steady policy stance is less critical. The fractional integration approach goes well beyond the I(0) stationary/I(1) non-stationary dichotomy to consider the possibility that variables may follow a long memory process. ‘Long memory’ means a significant dependence between observations widely separated in time, and, therefore, the effects of policy shocks may be temporary but long lasting.

2. Data: Sources and Description

We use annual data for the world’s total CO$_2$ emissions from fossil fuel consumption for the period ranging from 1751 to 2013. These data were obtained from the Carbon Dioxide Information Analysis Centre [Boden et al., 2013]. Aggregate CO$_2$ emissions are defined as a sum of five global CO$_2$ emissions components: CO$_2$ emissions from burning fossil fuels (solid, liquid, gas and gas flaring) and from cement production. The data do not consider emissions from land use, land-use change, and forestry, or even emissions from international shipping or bunker fuels. All variables are measured in million metric tonnes of carbon per year (Mt, hereafter) and were converted into units of carbon dioxide (CO$_2$) by multiplying the original data by 3.667. See Joint Research Centre of the European Commission (2014) for a detailed comparison among different available measurements of CO$_2$ emissions.

Over the past two and a half centuries, the world’s overall CO$_2$ emissions increased dramatically, rising from 11 Mt in 1751 to 36,131 Mt in 2013. The average annual rate of growth for the whole-period sample was 3.14%. For the more recent period of 2000-2013, the annual growth rate was 2.73%. Furthermore, for the last few years of the sample, due to mitigation efforts and, more importantly, to the economic and financial crisis, the slowdown in CO$_2$ emissions has been more pronounced, as the growth rate declined to under 2% in 2012 and 2013.
3. Fractional Integration

3.1 Fractionally-Integrated Processes

A fractionally-integrated process is a stochastic process with a degree of integration that is a fractional number, and with an autocorrelation function that exhibits persistence, albeit neither as an I(0) nor an I(1) process. Nevertheless, its persistence is consistent with a stationary process, where the autocorrelations decay hyperbolically. Because the autocorrelations die out slowly, the fractionally-integrated processes display long-run rather than short-term dependence, and for that reason are also known as long-memory processes [See, for example, Palma (2007)].

A time series, $x_t = y_t - \beta z_t$, where $\beta$ is the coefficients vector, $z_t$ represents all deterministic factors of the process $y_t$, and $t = 1, 2, ... n$ is said to be fractionally integrated of order $d$ if it can be represented by

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, 3, ...$$

(1)

where $L$ is the lag operator, $d$ is a real number that captures the long-run effect and $u_t$ is $I(0)$.

Through binomial expansion, the filter $(1 - L)^d$ provides an infinite-order $L$ polynomial with slowly and monotonically declining weights

$$(1 - L)^d = \sum_{j=0}^{\infty} \left( \begin{array}{c} d \\ j \end{array} \right) (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2!} L^2 - \frac{d(d-1)(d-2)}{3!} L^3 + ...$$

(2)

and thus (1) can be rewritten as:

$$x_t = dx_{t-1} - \frac{d(d-1)}{2} x_{t-2} + \frac{d(d-1)(d-2)}{3!} x_{t-3} + ... u_t.$$  

(3)

If $d$ is an integer, then $x_t$ is a function of a finite number of past observations. In particular, if $d = 1$, then $x_t$ is a unit root non-stationary process and, therefore, the effect of a random shock is exactly permanent. If $d = 0$, then $x_t = u_t$ and the time series is $I(0)$ and weakly auto-correlated (or dependent) with auto-covariances that decay exponentially over time. More formally,

$$\gamma_j = \alpha_1^j, \quad \text{for } j = 1, 2, ... \text{ and } |\alpha_1| < 1.$$  

(4)
Letting $d$ be a real number allows for a richer degree of flexibility in the specification of the dynamic nature of the series, and depending on the value of $d$ we can determine different levels of intertemporal dependency. In fact, when $d$ is a non-integer number, each $x_t$ depends on its past values way back in time. Moreover, the auto-covariance function satisfies the following property

$$\gamma_j \approx c_1 j^{2d-1}, \quad \text{for } j = 1, 2, \ldots \text{ and } 0 < |c_1| < \infty \quad (5)$$

where $\approx$ means that the ratio between the two sides of (5) will tend to unity as $j \to \infty$.

Assuming that the process $x_t$ has a spectral distribution such that the density function $f(\lambda)$ is given by

$$f(\lambda) = \left( \frac{\sigma^2}{2\pi} \right) \left| \frac{\theta(e^{-i\lambda})}{\phi(e^{-i\lambda})} \right|^2 \left[ 2(1 - \cos(\lambda)) \right]^{2d} \quad (6)$$

then, for low frequencies, as $\lambda \to 0^+$, we obtain

$$f(\lambda) \approx c_2 \lambda^{-2d} \quad (7)$$

where, $c_2 = \left( \frac{\sigma^2}{2\pi} \right) \left| \frac{\theta(1)}{\phi(1)} \right|^2 > 0$ and $\approx$ means that the ratio between the two sides of (7) will tend to unity as $\lambda \to 0^+$.

In general, the larger the value for the fractional-difference parameter $d$, the greater the degree of persistence. Specifically, we have several cases.

If $-0.5 < d < 0$, then the autocorrelation function decays at a slower hyperbolic rate but the process is called anti-persistent or, alternatively, to have rebounding behavior or negative correlation, because the autocorrelations for lags greater than zero are negative.

If $0 < d < 0.5$, the process $x_t$ reverts to its mean, but the auto-covariance function decreases very slowly and hyperbolically as a result of the strong dependence on past values. The spectral density function is unbounded at the origin and $x_t$ is said to exhibit long-memory behavior. This means that the effects of a random shock in the innovations of the series are transitory and the series will eventually revert to its
mean. Nevertheless, the effects will last longer than in the purely stationary case \((d = 0)\).

If \(0.5 < d < 1\), the process becomes more non-stationary in the sense that the variance of the partial sums (5) increases, but the series retains its mean-reverting property.

Finally, if \(d > 1\), the process is non-stationary and non-mean-reverting, i.e. the effects of random shocks are permanent.

### 3.2 ARFIMA Processes

An auto-regressive fractionally-integrated moving-average process, ARFIMA for short, is an extension of the traditional ARIMA model that allows for fractional degrees of integration. The autocorrelations of the ARFIMA process decay at a slower rate than the exponential rate associated with the ARMA process and, generally, with short memory processes. ARFIMA models were first introduced by Granger and Joyeux (1980) and Granger (1980, 1981) to solve problems with unit roots tests caused by either variable aggregation or the duration of shocks [see, again, Palma (2007)].

A process like (1) is called fractionally integrated of order \(d\) if \(d\) is a non-integer. If, in addition, \(u_t\) in (1) is an ARMA\((p, q)\), then \(x_t\) is an ARFIMA process and we have

\[
\phi(L^p)(1 - L)^d x_t = \theta(L^q) e_t
\]

where \(\phi(L^p)\) and \(\theta(L^q)\) are the polynomials of order \(p\) and \(q\) respectively, with all zeroes of \(\phi(L^p)\) and \(\theta(L^q)\) given, respectively, by

\[
\phi(z) = 1 - \phi_1 z - \phi_2 z^2 - \ldots - \phi_p z^p = 0
\]

\[
\theta(z) = 1 + \theta_1 z + \theta_2 z^2 + \ldots + \theta_q z^q = 0
\]

and lying outside the unit circle, withe\(e_t\) as white noise. Clearly, the process is stationary and invertible for \(-0.5 < d < 0.5\).

The estimation of the parameters of the ARFIMA model is done through maximum likehood. The log Gaussian likelihood was established by Sowell (1992b), and is

\[
\ell(\mathbf{y} | \hat{\eta}) = -\frac{1}{2} \left\{ T \log(2\pi) + \log | \mathbf{V} | + (\mathbf{y} - X \hat{\beta})' V^{-1} (\mathbf{y} - X \hat{\beta}) \right\}
\]
The covariance matrix $V$ has a Toeplitz structure:

$$
V = \begin{bmatrix}
\gamma_0 & \gamma_1 & \gamma_2 & \cdots & \gamma_{T-1} \\
\gamma_1 & \gamma_0 & \gamma_1 & \cdots & \gamma_{T-2} \\
\gamma_2 & \gamma_1 & \gamma_0 & \cdots & \gamma_{T-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\gamma_{T-1} & \gamma_{T-2} & \gamma_{T-3} & \cdots & \gamma_0
\end{bmatrix}
$$

where, $\gamma_0 = Var(y_t)$, and $\gamma_j = Cov(y_t, y_{t-1})$ for $j = 1, 2, \ldots, t - 1$ and $t = 1, 2, \ldots, T$.

### 3.3 ARFIMA Forecasting and Prediction-Accuracy Assessment

Given the symmetry properties of the covariance matrix, $V$ can be factorized as $V = LDL'$, where $D = \text{Diag}(v_t)$ and $L$ is lower triangular, so that:

$$
L' = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
\tau_{1,1} & 1 & 0 & \cdots & 0 \\
\tau_{2,2} & \tau_{2,1} & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\tau_{(T-1),(T-1)} & \gamma_{(T-1),(T-2)} & \tau_{(T-2),(T-3)} & \cdots & 1
\end{bmatrix}
$$

Moreover, let $\tau_t = V_t^{-1}y_t$, $y_t = (y_1, y_2, \ldots, y_T)'$ and $V_t$ is the $t \times t$ upper left sub-matrix of $V$.

Let $z_t = y_t - x_t \hat{\beta}$. The best linear forecast of $z_{t+1}$ based on $z_1, z_2, \ldots, z_t$ is

$$
\hat{z}_{t+1} = \sum_{k=1}^{t} \tau_{t,k} z_{t-k+1}
$$

Moreover, the best linear predictor of the innovations is $\hat{\epsilon} = L^{-1}z$, and the one-step-ahead forecasts for $\hat{y}$, in matrix notation, is

$$
\hat{y} = L^{-1}(y - X\hat{\beta}) + X\hat{\beta}.
$$

The testing for the accuracy of predictions is based on the average squared forecast error, which is computed as

$$
MSE(z_{T+k}) = \hat{\gamma}_0 - \gamma_k' V^{-1} \hat{\gamma}_k, \text{ where } \hat{\gamma}_k = (\hat{y}_{T+k-1}, \hat{y}_{T+k-2}, \ldots, \hat{y}_k).
$$

There is a wide diversity of loss functions available and their properties vary extensively. Even so, all of these share a common feature, in that “lower is better.” That is, a large value indicates a poor forecasting performance, whereas a value close
to zero implies an almost-perfect forecast. We use three average loss indicators: the Mean Absolute Percentage Error (MAPE), the Adjusted Mean Absolute Percentage Error (AMAPE), and the U-statist inequality coefficient.

The MAPE and the AMAPE are relative measures, in that they are percentages. In particular, the MAPE may be interpreted as the percentage error, and has the advantage of being bounded from below by zero. Therefore, the lower the indicator the greater the model’s forecast accuracy. Nevertheless, this loss function has drawbacks in any practical application. First, with zero values, we have a division by zero issue. Second, the MAPE does not have an upper limit. The AMAPE corrects almost completely the asymmetry problem between actual forecast values, and has the advantage of having both a zero lower bound and an upper bound. Like the MAPE, the smaller the AMAPE, the greater the accuracy of predictions made.

The U-statistic provides a measure of how well a time series of estimated values compares to a corresponding time series of observed values. The Theil inequality coefficient lies between zero and one, with zero suggesting a perfect fit. It can be decomposed into three sources of inequality; bias, variance, and covariance proportions coverage. The bias component of the forecast errors measures the extent to which the mean of the forecast is different from the mean of the recorded values. Similarly, the variance component tells us how far the variation of the forecast is from the variation of the actual series. Finally, the covariance proportion measures the remaining unsystematic component of the forecasting errors. As expected, the three components add up to one.

4. The Basic Empirical Results

4.1 Model Specification

Estimation results for the ARFIMA(p, d, q) model are presented in Table 1. The best specification was chosen using the Schwartz Bayesian Information Criterion (BIC) and includes two statistically-significant autoregressive terms, of first and second order, and two statistically-significant moving-average terms, of first and seventh order. Furthermore, global CO₂ emissions are fractionally integrated with a statistically significant degree of persistence of d = 0.354. The confidence intervals for the
estimated fractional integration parameters are narrow, in the positive range, and lower than 0.5. This means that, the series are better characterized as ‘stationary, but with long memory’. The effects of a one-time random shock in the innovations of these series are transitory, as the series are mean reverting. The effects of the one-time random shocks, however, will last longer than in the purely stationary case.

4.2 In-Sample Global CO₂ Emissions Forecasts

Figure 1 plots the actual values against the in-sample forecasts for global CO₂ emissions between 1900 and 2013 (in-sample forecasts between 1750 and 1899 are available upon request). Table 2 summarizes our forecasting accuracy analysis for the in-sample predictions. For the whole sample period, the mean absolute percentage error, MAPE, is 4.64% while the adjusted mean absolute percentage error, AMAPE, is 5.9%, indicating a good forecast performance. Moreover, only 7.8% of the predicted values lie outside the 95% confidence interval. In particular, 4.7% are above the upper limit while 3.1% are below the lower limit. In turn, the U-statistic shows a low level of inequality, suggesting that the forecasts compare very well with actual CO₂ emissions. Moreover, the forecasts are unbiased and have small variance proportion. Accordingly, most of the forecast error, 95.48%, can be attributed to the covariance component, i.e., the unsystematic forecasts error.

4.3 Out-of-Sample Forecasts of Global CO₂ Emissions

Using data from 1751 to 2008 to estimate the ARFIMA model, we forecast the remaining 5 observations until 2013. The results are shown in Fig. 2 and in Table 3. The forecast is acceptable for the sub-sample, and only somewhat overestimates the emissions for 2009 and underestimates CO₂ emissions for 2010. It is worth highlighting that the recorded CO₂ emissions in 2009 fell below the lower limit of the 95% confidence interval. For the remaining three years, the forecasts are fairly aligned with the CO₂ emissions on record. The MAPE of the forecast is only 1.08%, and the same occurs when we correct the asymmetry problem between recorded and forecast values. Indeed, the AMAPE of the forecast is just 0.54%. The U-statistic, the bias proportion and the variance proportion are all small, indicating a very good forecast performance, with 99.46% of the bias concentrated on the unsystematic forecasts error.
5. **Global CO₂ Emissions Forecasts Until 2100**

5.1 **The ARFIMA Forecasts**

Having established a good forecasting performance of the statistical ARFIMA model in both in-sample and hangout sample forecasts, we use these estimates to forecast CO₂ emissions until 2100. The results are shown in Fig. 3 and in Table 4. CO₂ emissions are projected to increase from 36,131 Mt in 2013 to almost 51,883 Mt in 2100. More specifically, the levels of CO₂ emissions in 2030, 2040, 2050 and 2100 are about 27.4%, 34.4%, 39.8%, and 52.9% above their respective 2010 levels.

A notable feature of our forecast is that it suggests a continuous slowdown in the increase in the CO₂ emissions. The ARFIMA framework seems to capture quite well the shift in the CO₂ emissions pattern that actually occurred in the last decade due to more active environmental policies worldwide, as well as the more recent financial crisis and its economic aftershocks.

Despite this pattern, our projection suggests a reference path for global CO₂ emissions which, unsurprisingly, does not meet the global CO₂ emission goals designed to stabilize the concentration of CO₂ in the atmosphere at levels consistent with avoiding the most extreme outcomes of climate change.

5.2 **Comparing the Forecasts with Other Available Reference Forecasts**

Next, we compare the forecasts we made with the reference forecasts used by different international organizations. To facilitate comparisons, we focus on projected reference changes at different points vis-à-vis the CO₂ emission levels in 2010. IEA (2007) projects reference CO₂ emissions in 2030 and 2050 at 39% and 48-55% above emission levels in 2010. In turn, OECD (2012) projects 2030, 2040, 2050 reference emissions about 32%, 58% and 70% above 2010. Finally, EIA (2013) projects 2030 and 2040 reference emissions at 33.5% and 42.9% above 2010 levels.

As a reminder, we project 2030, 2040, 2050 CO₂ reference emissions at about 27.4%, 34.4%, 39.8%, above 2010 emission levels, respectively. This suggests that we project a less pessimistic trend of CO₂ emissions into the future. Furthermore, given the slowly declining pattern of marginal changes in CO₂ emissions that we project, the difference
between our reference case and the existing reference projections will tend to widen as we move further into the future.

5.3 Comparing Our Forecasts with Alternative Policy Scenarios for CO₂ Emissions

The relevance of our lower reference case projections becomes apparent when we consider the paths for CO₂ emissions projected under alternative policy scenarios or required to reach certain emission targets.

For example, the European Union aims to achieve by 2030 a reduction of CO₂ emissions that is equivalent to 40% of 1990 emission levels [see, for example, EC (2014)]. Translating this goal into a global objective would mean a reduction of about 9Mt by 2030, compared to the reference scenario. On the other hand, IEA (2015)’s INDC scenario aims at an 8% increase in 2030 versus 2013, or an increase of about 3Mt. Other scenarios such as IEA (2015)’s Scenario 450 assume an actual reduction in emissions, while a bridge scenario postulates seriously curtailing emissions.

Under our reference forecast, the globalized EU emissions target would mean a reduction about the same size of the increase we project for 2030 compared to 2010, i.e. 9.2 Mt. In practice, this target would mean keeping emissions at the 2010 level, an effort worth a 25% reduction in emissions under our reference scenario, and between 32% and 39% under alternative reference scenarios. In turn, for the IEA (2015) INDC scenario’s objectives, given our reference scenario, this would mean cutting projected additional CO₂ emissions by an additional 33%. The point is that, with our lower projections of CO₂ emissions, the ongoing policy goals are much more attainable.

6. Conclusions and Policy Implications

In this note we developed alternative new reference forecasts for CO₂ emissions. The reference forecast is based on an ARFIMA model, estimated using global CO₂ emissions data from 1750 to 2013. These new reference forecasts are free from the additional assumptions on demographic and economic variables that are commonly used in most reference forecasts, as they only rely on the intrinsic properties of the stochastic process for CO₂ emissions and all the observed information it incorporates. In this sense they are much closer to their fundamentals.
Our reference forecast suggests that CO₂ emissions are clearly below the levels suggested by other reference scenarios available in the literature. This is important, as it suggests that the ongoing policy goals are actually much closer to reach than what is often implied by the standard CO₂ reference emission scenarios. This is not to say that policy makers should rest on their laurels and relax their ongoing energy and environmental policy efforts to achieve such targets, but rather that whatever efforts are agreed upon will likely be more effective than what is implicit in the existing reference forecasts. In other words, the current CO₂ emissions targets can be achieved at a much lower overall cost than what is suggested by the conventional reference forecasts.

A possible reaction to the lower emissions path in the new reference scenario presented here could be that it is better to err on the side of caution – that is, it would be better to use the higher projected CO₂ emissions as suggested by other reference forecasts. However, this situation reflects a delicate trade-off between two competing negotiation strategies. On one hand, there is a view that having higher emission reference forecasts increases the likelihood of decisive policy action and thus guarantees that the policies adopted will less likely undershoot the required objectives. On the other hand, there is also the view that, having lower and more realistic reference emissions projections gives a more accurate assessment of the policy efforts that are indeed required, and thus underscores the lower costs involved in mitigation efforts, thereby making all the more likely the adoption of more widespread energy and environmental policy efforts.

Finally, it should be mentioned that we see our new reference forecasts as a benchmark to be compared and contrasted with other reference forecasts generated by either univariate or more complex multivariate models. It ought to be a natural starting point to which other economic or demographic assumptions are to be added upon. Indeed, the evidence for long-term memory in CO₂ emissions that we uncovered through our fractional integration approach suggests that transitory shocks will have temporary although long-lasting effects. Accordingly, in forecasting alternative and more elaborated policy scenarios, a more comprehensive multivariate approach should be pursued.
References


<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Estimates</th>
<th>Std. Err.</th>
<th>95% Confidence interval</th>
<th>BIC</th>
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</thead>
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<tr>
<td></td>
<td>$\alpha_1$</td>
<td>0.444</td>
<td>0.110</td>
<td>[0.228 ; 0.629]</td>
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<td></td>
<td>$\alpha_2$</td>
<td>0.552</td>
<td>0.109</td>
<td>[0.339 ; 0.768]</td>
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<td>Global CO$_2$ emissions</td>
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<td>0.082</td>
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<td>$d$</td>
<td>0.354</td>
<td>0.046</td>
<td>[0.263 ; 0.444]</td>
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</table>

Note: $\hat{\alpha}$ stands for the estimated value of the parameter associated with $x_{t-p}$ of the AR,$\hat{\theta}$ stands for the estimated value of the stochastic term of order $q$ ($e_{t-q}$) of the MA component, and $p$-values are in brackets.
## Table 2– Five-Year In-Sample Forecasts for the Horizon: 1900 to 2013

<table>
<thead>
<tr>
<th>Time</th>
<th>Actual</th>
<th>In-sample forecasts</th>
<th>Innovations</th>
<th>95% Confidence interval</th>
</tr>
</thead>
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<td></td>
<td></td>
<td>Level</td>
<td>Percentage of the actual</td>
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<td>1,956.6</td>
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<td>1.39</td>
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<td>6,990.3</td>
<td>491.6</td>
<td>6.57</td>
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<tr>
<td>1960</td>
<td>9,412.8</td>
<td>9,222.4</td>
<td>190.4</td>
<td>2.02</td>
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<tr>
<td>1965</td>
<td>11,468.3</td>
<td>11,292.0</td>
<td>176.3</td>
<td>1.54</td>
</tr>
<tr>
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<td>14,850.2</td>
<td>14,395.4</td>
<td>454.8</td>
<td>3.06</td>
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<tr>
<td>1975</td>
<td>16,839.7</td>
<td>17,344.8</td>
<td>-505.1</td>
<td>-3.00</td>
</tr>
<tr>
<td>1980</td>
<td>19,474.2</td>
<td>20,352.3</td>
<td>-878.0</td>
<td>-4.51</td>
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<tr>
<td>1985</td>
<td>19,928.5</td>
<td>19,586.5</td>
<td>342.0</td>
<td>1.72</td>
</tr>
<tr>
<td>1990</td>
<td>22,449.3</td>
<td>22,629.5</td>
<td>-180.2</td>
<td>-0.80</td>
</tr>
<tr>
<td>1995</td>
<td>23,442.3</td>
<td>23,392.2</td>
<td>50.1</td>
<td>0.21</td>
</tr>
<tr>
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<td>24,787.0</td>
<td>24,254.1</td>
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<tr>
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<td>29,652.8</td>
<td>29,310.2</td>
<td>342.6</td>
<td>1.16</td>
</tr>
<tr>
<td>2010</td>
<td>33,587.9</td>
<td>32,690.4</td>
<td>897.5</td>
<td>2.67</td>
</tr>
<tr>
<td>2013</td>
<td>36,131.0</td>
<td>36,169.8</td>
<td>-38.8</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

**Mean Absolute Percentage Error (MAPE)**: 5.90%

**Adjusted Mean Absolute Percentage Error (AMAPE)**: 2.90%

**Theil Inequality Coefficient**: 0.0113

**Mean Squared Error decomposition**

| Bias proportion | 0.0137 |
| Variance proportion | 0.0240 |
| Covariance proportion | 0.9548 |
Figure 1- In-Sample ARFIMA Predictions for 1900-2013
### Table 3–Forecasts and Actual CO₂ Emissions for the Hangout Sample: 2009-2013

<table>
<thead>
<tr>
<th>Time</th>
<th>Actual</th>
<th>In-sample forecasts</th>
<th>Innovations</th>
<th>95% Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Level</td>
<td>Percentage of the actual</td>
</tr>
<tr>
<td>2009</td>
<td>32,023.4</td>
<td>32,750.1</td>
<td>-726.7</td>
<td>-2.27</td>
</tr>
<tr>
<td>2010</td>
<td>33,587.9</td>
<td>32,690.4</td>
<td>897.5</td>
<td>2.67</td>
</tr>
<tr>
<td>2011</td>
<td>34,651.4</td>
<td>34,596.6</td>
<td>54.8</td>
<td>0.16</td>
</tr>
<tr>
<td>2012</td>
<td>35,425.2</td>
<td>35,499.6</td>
<td>-74.4</td>
<td>-0.21</td>
</tr>
<tr>
<td>2013</td>
<td>36,131.0</td>
<td>36,169.8</td>
<td>-38.8</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

**Mean Absolute Percentage Error (MAPE)** 1.08%

**Adjusted Mean Absolute Percentage Error (AMAPE)** 0.54%

**Theil Inequality Coefficient** 0.0150

**Mean Squared Error decomposition**
- **Bias proportion** 0.0019
- **Variance proportion** 0.0037
- **Covariance proportion** 0.9944
Figure 2 – Forecasts and Actual CO₂ Emissions for the Hangout Sample: 2009-2013
<table>
<thead>
<tr>
<th>Years</th>
<th>Total CO₂ emissions (forecasts - fᵢ)</th>
<th>RMSE</th>
<th>95% Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MtCO₂</td>
<td>RMSE/ᵢ (%)</td>
<td>Lower limit</td>
</tr>
<tr>
<td>2015</td>
<td>37,274.1</td>
<td>545.4</td>
<td>1.5</td>
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<tr>
<td>2020</td>
<td>39,545.1</td>
<td>1,258.9</td>
<td>3.2</td>
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<tr>
<td>2025</td>
<td>41,321.5</td>
<td>2,098.3</td>
<td>5.1</td>
</tr>
<tr>
<td>2030</td>
<td>42,789.8</td>
<td>2,886.3</td>
<td>6.7</td>
</tr>
<tr>
<td>2035</td>
<td>44,044.1</td>
<td>3,628.9</td>
<td>8.2</td>
</tr>
<tr>
<td>2040</td>
<td>45,135.6</td>
<td>4,335.5</td>
<td>9.6</td>
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<tr>
<td>2045</td>
<td>46,097.2</td>
<td>5,012.1</td>
<td>10.9</td>
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<tr>
<td>2050</td>
<td>46,949.2</td>
<td>5,663.0</td>
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<tr>
<td>2055</td>
<td>47,710.5</td>
<td>6,291.1</td>
<td>13.2</td>
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<tr>
<td>2060</td>
<td>48,392.7</td>
<td>6,898.8</td>
<td>14.3</td>
</tr>
<tr>
<td>2065</td>
<td>49,004.5</td>
<td>7,487.9</td>
<td>15.3</td>
</tr>
<tr>
<td>2070</td>
<td>49,555.3</td>
<td>8,059.7</td>
<td>16.3</td>
</tr>
<tr>
<td>2075</td>
<td>50,053.5</td>
<td>8,615.5</td>
<td>17.2</td>
</tr>
<tr>
<td>2080</td>
<td>50,499.7</td>
<td>9,156.3</td>
<td>18.1</td>
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<tr>
<td>2085</td>
<td>50,907.6</td>
<td>9,683.0</td>
<td>19.0</td>
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<tr>
<td>2090</td>
<td>51,272.1</td>
<td>10,196.4</td>
<td>19.9</td>
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<tr>
<td>2095</td>
<td>51,595.1</td>
<td>10,697.1</td>
<td>20.7</td>
</tr>
<tr>
<td>2100</td>
<td>51,883.3</td>
<td>11,185.7</td>
<td>21.6</td>
</tr>
</tbody>
</table>
Figure 3–Global CO₂ Emissions Forecasts from 2013 until 2100