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Abstract

This paper models volatility in CO2 EUAs emission returns using a FIGARCH approach. Our findings overwhelmingly suggest that conditional variance in CO2 emissions allowance returns is stationary and mean reverting autocorrelations decaying at a hyperbolic rate. Hence, a shock to forecast of future conditional variance will be temporary but it will last longer.
Our results have important policy implications, as the knowledge of the stochastic properties of the conditional variance is of particular relevance for decisions on investment in abatement activities, for the design of arbitrage strategies to take advantage of momentary opportunities in energy markets. Moreover, our results also suggest the importance of accounting for the interactions of volatility in the EUAs CO2 emissions market with energy sectors, the economy, and climate, both in terms of modeling and forecasting.

Keywords: CO2 emission prices, volatility, FIGARCH

JEL Classification: C22, E52, E58, F30, G10

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Volatility in CO_2 EUAs returns: a FIGARCH approach

1. Introduction

The aim of this study is to use an ARMA-FIGARCH model to analyze and measure the presence of long memory in the volatility of returns associated with the price of European Union Allowances (EUAs hereafter) in the European Union Emissions Trading System (EU ETS hereafter).

Established in 2005 by Directive 2003/87/EC, the EU ETS was mainly aimed at the energy production sector and industrial processes (namely the ferrous metal, mineral, refining and paper pulp industries) and brought about an important change from the traditional GHG environmental policy which had basically been supported by command-and-control instruments. It brought a shift of emphasis to a strategy which was based on the decision-making process for the allocation of resources used by companies, and sought to achieve two important objectives under an emissions cap-and-trade regime: a) to provide incentives for industry to reduce GHG emissions to the desired levels; and b) to contribute to advancing the implementation of low-carbon technologies and energy efficiency. Currently, the EU ETS market is held to have prime responsibility for the 22.5% reduction in total GHG emissions by the 28 EU Member States between 1990 and 2014 [see, among others, Zhang & Wei (2010)].

The literature on the volatility of returns associated with the EU ETS market concentrates principally on the factors that impact variation in price, and is still very focused on the pilot phase of this market. Some of these factors arise from the supply side and others from the demand side of emissions allowances. On the supply side, it has been suggested that both the fairly stringent level and nature of the cap and also the rules for its allocation by the ETS sectors and over time hold responsibility for the volatile nature of this new market; it may also have caused undesirable effects on distribution that may distort competitiveness [see, for example, Neuhoff et al. (2006), Kattner et al. (2007), Woerdman (2008) and Clò (2010)]. Similarly, the discrepancy between the amount set by the cap and actual emissions [see, for example, Ellerman and Buchner (2008)] as well as advance information disclosure about each country's (short or long) position on allowances [Alberola et al. (2008) and Alberola et al. (2009)], help explain some episodes of instability in the EU ETS, especially in its pilot phase.

Parsons at al. (2009) and Helm (2009) also suggest that other factors may be at the root of high private costs for ETS sectors, thus contributing to the risk of carbon leakage. These include: the existence of multi-annual periods with fixed endpoints; the time profile of the annual placement of the cap; the allowance allocation scheme on the primary market – grandfathering rather than auctioning; and the presence of a slight possibility (or even an absence of it) for inter-temporal flexibility in complying with the cap by allowance banking and borrowing over time. Finally, the European Commission itself recognizes that only an auctioning system can ensure environmental and economic efficiency and avoid distorting competition and producing undesirable effects on distribution [SEC 2008].

On the demand side, there is strong evidence in the literature of the impact that the price of fossil fuels and energy – particularly electricity – has on the stability of the prices of CO2 emissions allowances [Mansanet-Bataller et al. (2007) and Oberndorfer (2009)]. In addition, empirical evidence suggests a close relationship between the price of allowances and the behavior of financial (stocks and bonds) markets. Indeed the suggestion is that market dynamics are influenced by the economic cycle [see Chevallier (2009) and Chevallier (2011c)].

Some authors also suggest that atmospheric conditions and extreme phenomena (related to temperature, availability of water and wind speed) have caused significant shocks in the carbon market [Mansanet-Bataller et al. (2011), Alberola et al. (2008) Hintermann (2010)] and Chevallier (2011a)]. Finally, some studies have shown that financial crises, increases in the use of renewable energy sources and negotiations on international climate change can potentially influence price stability in the carbon market [see, among others, Tang and Xiong (2009), Blyh and Bunn (2011) and Chevallier, (2011a)].

There is now a budding field of literature specifically focused on the stochastic properties of CO2 prices and returns, and providing evidence for heteroskedasticity in the conditional variance in daily CO2 returns [see Paolella and Taschinin (2008), Benz and Trück (2009), Feng et al. (2011), Conrad (2012) and Liu and Chen (2013)].

This article contributes to expanding and enriching the debate in the literature on the modeling and explanation of EUA price movements. In particular, our work adds to the

very small set of approaches which assume that current prices embrace all the information on the key factors that determine the level and variability of prices in the EU ETS in order to assess and measure the presence of long memory in the process of generating data for the carbon price time series. We do so by testing for fractional integration using an $ARMA(p,q) - FIGARCH(p,d,)$ model. An $ARMA - FIGARCH$ model is a generalization of the traditional $ARMA - GARCH(p,q)$ class of models, which frees it from the $I(0)/I(1)$ dichotomy, and therefore allows an estimation of the degree of integration of the conditional variance. The intensity of this phenomenon is measured by the coefficient of fractional integration "d." Long memory implies a significant dependence between observations which are widely separated in time, and therefore the effects caused by shocks tend to decay slowly, although still mean-reverting in nature. Fractional integration models were introduced by Granger and Joyeux (1980), Granger (1980, 1981), Sowell (1992a, b), Baillie (1996), and Palma (2007) and provide greater flexibility in assessing the characteristics of time series. Similarly to $ARFIMA$ models in the case of the conditional mean, the effect of a shock on conditional variance in a stochastic Fractionally Integrated Generalized AutoRegressive Conditionally Heteroscedastic (FIGARCH) process is temporary in the sense that the effects on values of future conditional variance will tend to dissipate at a hyperbolic regression rate.

The remainder of this article is organized as follows. Section 2 provides a brief technical description of the methodology used. Section 3 presents the data set and some description of the data. Section 4 discusses the empirical findings, first considering the traditional unit roots approach and then using our fractional integration approach. Finally, Section 5 provides a summary of the results, and discusses their implications for energy and environmental policies.

2. Volatility in CO2 prices: methodological framework

This section presents the basic framework of some of the Generalized AutoRegressive Conditionally Heteroscedastic (GARCH) models and the FIGARCH model which we use to assess the volatility in CO2 emissions allowance prices.

2.1 The class of GARCH models

The class of GARCH models has been one of the most widely used methods for modeling and forecasting volatility. The simplest specification of a GARCH model is [Bollerslev (1986)]

$$y_t = \mu + \sum_{i=1}^{p} \rho_i y_{t-i} + \sum_{j=1}^{q} \lambda_j u_{t-j} \tag{1}$$

$$h_t = \omega + \sum_{i=1}^{s} \alpha_i u_{t-i}^2 + \sum_{j=1}^{r} \beta_j h_{t-j} \tag{2}$$

Eq.(1) is the conditional mean – which describes how the dependent variable, y_t, varies over time – and can take any form. Eq. (2) is the conditional variance equation and is a weighted function of a long-term mean (ω), information about past volatility (the ARCH terms, $\sum_{i=1}^{s} \alpha_i \, u_{t-i}^2$) and the past fitted variance from the model (the GARCH terms, $\sum_{j=1}^{r} \, \beta_j \, h_{t-j}$). It should be noted that $h_t = \sigma^2$ is the conditional variance of variable y_t as it is a one-period-ahead forecast variance with its calculation based on past relevant information. In addition, $u_t = z_t . \sqrt{h_t}$ is the squared residual from the mean equation. The standardized innovations z_t have zero conditional mean and unit conditional variance by construction. More formally, $E_{t-1}(z_t) = 0$ and $E_{t-1}(z_t^2) = 1$.

By definition, the conditional variance must be strictly positive. Since all terms on the RHS of Eq.(2) are positive (they are all squared and lagged errors), a sufficient condition – though not necessary for this non-negativity to occur – is that all the coefficients must be non-negative as well; $\omega \geq 0, \alpha_i \geq 0$ and $\beta_j \geq 0$ for $i(j) = 1, \dots, s(r)$. In addition, the long-run (unconditional) variance is given by

$$var(u_t) = \frac{\omega}{[1 - (\alpha_1 + \beta_1)]} \tag{3}$$

Since the conditional variance changes over time it will converge on the unconditional variance only if $\alpha_1 + \beta_1 < 1$. In other words, $\alpha_1 + \beta_1 < 1$ measures the degree of persistence in volatility, in which case the unconditional variance is said to be stationary

in variance. However, when $\alpha_1 + \beta_1 \geq 1$, the unconditional variance is not defined, and for that reason this case is termed non-stationary in variance.

A property common to most volatile processes is that bad news and good news (captured by the sign of u_t) of the same magnitude may cause different effects in the magnitude of volatility. In particular, bad news ($u_t < 0$) is likely to cause volatility to rise more than good news of the same magnitude. This effect is attributed to leverage effects or to volatility-feedback mechanisms. The asymmetry hypothesis can be tested within the GARCH framework simply by adding a term which distinguishes positive news from negative news. The conditional variance is then given by

$$h_t = \omega + \sum_{i=1}^{s} \alpha_i u_{t-i}^2 + \sum_{j=1}^{r} \beta_j h_{t-j} + \sum_{k=1}^{v} \gamma_k u_{t-k}^2 I_{t-k} \tag{4}$$

where $I_t = 1$ if $u_t < 0$ and $I_t = 0$ otherwise.

The threshold $GARCH$ ($TGARCH$, hereafter) as this model is known, was introduced independently by Zakoïan (1994) and Glosten et al. (1993) and for that reason it is also known as GJR. If $\gamma_i \neq 0$, the impact of the news is asymmetric. In particular, a negative shock ($u_t < 0$) will have an impact of $\sum_{i=1}^{s} \alpha_i + \sum_{k=1}^{v} \gamma_k$ (for $i = k$) on the conditional variance, while a positive shock ($u_t > 0$) will have an impact of $\sum_{i=1}^{s} \alpha_i$ on the conditional variance. Accordingly, when $\sum_{k=1}^{v} \gamma_k > 0$ (or $\sum_{k=1}^{v} \gamma_k < 0$), bad (good) news increases volatility more than good (bad) news, and therefore there is a leverage effect of order *v-th*. In the TGARCH model the necessary conditions for non-negativity need to be complemented by an additional condition involving the asymmetric parameter $\gamma_i : \alpha_i + \gamma_k \geq 0$ for $i = k$.

Another extension of the basic $GARCH$ model is the Exponential $GARCH$ model ($EGARCH$, hereafter) proposed by (Nelson (1991), which uses the $\log(\sigma^2)$ instead of the level of the conditional variance. The specification of the conditional variance is:

$$\log(h_t) = \omega + \sum_{i=1}^{s} \alpha_i \left| \frac{u_{t-i}}{\sigma_{t-i}} - E\left(\frac{u_{t-i}}{\sigma_{t-i}}\right) \right| + \sum_{j=1}^{r} \beta_j \log(h_{t-j}) + \sum_{k=1}^{v} \gamma_k \frac{u_{t-k}}{\sigma_{t-k}} \tag{5}$$

Given that the GARCH component is the $\log(\sigma^2)$, there is no need to artificially impose non-negative constraints, as the conditional variance will be positive even if the

parameters are negative. Moreover, the EGARCH model includes an asymmetry term γ_k, which will be negative if the relationship between volatility and the standardized innovations is negative. Note that the asymmetric components in both TGARCH and EGARCH models do not match. Indeed, the parameter γ_k must be given an opposite interpretation in both models. In the TGARCH model, when $\gamma_k > 0$, bad news increases volatility, while in EGARCH, the leverage effect is present if $\gamma_k < 0$.

2.2 The IGARCH and the FIGARCH models

One of the characteristics which the results of empirical applications of the previous models have in common was the presence of persistence in the estimated conditional variance [see Engle (1982), among others]. This finding led Engle and Bollerslev (1986) to introduce what they termed the Integrated GARCH model (or IGARCH). This model is to the covariance-stationary GARCH model class as $I(1)$ processes (1) are to $I(0)$ processes in the case of the conditional mean. IGARCH models impose a constraint such that the sum of the ARCH and GARCH components must be one. In the specific case of an $IGARCH(1{,}1)$ model, it must be ascertained that $\alpha_1 + \beta_1 = 1$, and the process becomes a unit root in variance.

It should be noted that the distinction between a stationary process and an integrated process for conditional variance is as limited as the distinction between an $I(1)$process and a $I(0)$ process in the case of the conditional mean. And, as in stochastic processes for the conditional mean, the traditional dichotomy $I(0)/I(1)$ can be eliminated if the degree of integration can be a fractional number, $I(d)$. Contrary to what occurs in an $I(0)$, process, where a shock dissipates at an exponential rate, and in a $I(1)$ process, where there is no reversion to the mean, a shock affecting an $I(d)$ stochastic process, where $0 < d < 1$, tends to dissipate at a hyperbolic rate.

The $FIGARCH(p, d, q)$ process can be represented in the traditional way by the conditional variance equation [see Bollerslev and Mikkelson (1996) and Baillie et al. (1996)]:

$$[1 - \beta(L)]\sigma_t^2 = \omega + [1 - \beta(L) - \phi(L)(1 - L)^d\, u_t^2] \qquad (6)$$

where L represents the phase shift operator, $\phi(L)=[1-\alpha(L)-\beta(L)]$, $\alpha(L)=\alpha_1 L+\cdots+\alpha_q L^q$ and $\beta(L)=\beta_1 L+\cdots+\beta L^p$ are the phase shift polynomials. In addition, it is assumed that the $ARMA(p,q)$ roots of the polynomials $\phi(L)$ and $[1-\beta(L)]$ are within the unit circle. In addition, as with the $GARCH(p,q)$, process, which can be represented as an $ARMA(p,q)$ process where $\{u_t^2\}$ is the conditional mean, a $FIGARCH(p,d,q)$ process can also be represented by an $ARFIMA(p,d,q)$ process where $\{u_t^2\}$ is the conditional mean,

$$\phi(L)(1-L)^d\, u_t^2=\omega+[1-\beta(L)]v_t \qquad (7)$$

with $v_t\equiv u_t^2-\sigma_t^2$. In this class of FIGARCH models, the short-term behavior of the time series is captured by the ARMA components, while the long-run dependence is captured by the fractional integration coefficient d [Sowell (1992a)].

The estimation strategy used to test the presence of long memory in the rate of CO2 allowance returns is based on the methodology put forward by Bollersev and Mikkelsen (1996), and involves the use of two models, $ARMA(2{,}0)-IGARCH$ and $ARMA(2{,}0)-FIGARCH(1,d,1)$. Taking the daily frequency of the data into account, we can cater for the impact that non-transaction days have on the variance in rate of return on the first day after a break in trading. We do this by including the variable "gap_t" for the number of days without allowance transactions between time t and time t-1 [see, for example, Baillie and Bollerslev (1989)]. As a consequence, the new variable models the impulse response caused by the weekend market close $h_t=\omega(1-\beta(1))^{-1}+\lambda(L)\{u_t^2-\delta gap_t\}+\delta gap_t$. In particular, the general model $ARCH(1{,}1)$ takes the form $h_t=\omega(1-\beta(1))^{-1}+\lambda(L)\{u_t^2-\delta gap_t\}+\delta gap_t$ [with $\lambda(L)=1-\left(1-\beta_1 L\right)^{-1}\left(1-\phi_1 L\right)(1-L)^d$) and $\phi_1=\alpha_1+\beta_1$] and captures two effects; first, the instantaneous effect on the conditional variance h_t and second, the expected change to volatility for the day following the halt in trading, h_{t+1}. Take the example of Monday = t, with $gap_t=2$ and $gap_{t-1}=0$. The instant effect of a halt in trading makes the volatility of that day's session vary $\Delta h_t=\delta$, which in turn affects the expected volatility of Tuesday at the value $\Delta h_{t+1}=(d+\phi_1)\delta$. Empirical evidence suggests that the variance in returns tends to be higher on Mondays, although this increase is smaller the longer the non-

transaction period extends [see French and Roll (1986)] and Bollerslev and Mikkelsen (1996), among others)], whereby it is expected that $\delta > 0$.

3. Data and stationary analysis

This section describes the basic data set and presents the results of the unit root analysis.

3.1 Data: sources and description

Our data set consists of daily CO2 equivalent (CO2e, hereafter) close prices on EU ETS, from January 2, 2008 through May 23, 2014, for a total of T = 1,628 observations. Data was obtained from EEX. Following the usual practice, we transform daily CO2e prices into a daily returns series $y_t = 100.\ln(p_t/p_{t-1}),\ t = 1, \dots, T$. The time subscript t refers to trading days. Daily data are also available for the first EU ETS period (between January 2005 and December 2007), but due to the pilot nature of this phase, they were not considered in this study. Figure 1 plots the daily prices for CO2e emissions and Figure 2 plots daily returns. Table 1 presents some summary statistics, where μ stands for the mean, σ stands for the standard deviation and σ/μ stands for the coefficient of variation. The coefficient of variation shows the extent of variability in relation to the mean.

In general, the average price per ton of CO2e for our sample was €12.08, with a mean standard error of 0.150 and a variability coefficient of 1.25% (see Table 1). It can also be seen in Table 1 that the price per ton of CO2e dropped consistently over the sample period from a high of €29.20 on 1 April 2008 to €2.78 on 14 July 2013 (see Figure 1).

The daily rate of price change reveals a completely different time pattern. Indeed, the average daily value of returns (as a percentage) during the sample period was -0.041%. Conversely, the average standard error (0.078%) is about twice the average, which suggests great variability in the returns associated with CO2e emissions allowances over the sample period. It is noteworthy that the years 2008, 2011 and 2014 had the highest average daily volatility in price, while the critical years for returns were, 2013, 2012, 2009 and 2014, in this order. Meanwhile, the highest percentage increase in CO2e price was 22.87% on April 17, 2013 and the largest daily percentage decrease was 24.87% three weeks later, on May 5 (see Figure 2). These variations were certainly a result of

the combination of several factors that came about in 2013, in April in particular. In fact, 2013 saw the start of accounting of aviation emissions, and the first endpoint for the accounts of these emissions and any compensation for companies in EUAs was in April. This caused great apprehension in the market, which led the European Commission to delay the start of operations of this new legal framework. In addition, that same month, the European Parliament had also scheduled a debate and vote on a proposal from some member states to reduce that year's cap by 900 million tons of CO2e in order to raise the price of CO2e, which made the EUA market highly unstable.

[Insert Table 1, figure 1 and 2 around here]

4. Volatility in CO2e prices : Empirical results

This section presents the model specifications which describe both the conditional mean and the conditional variance in the rate of CO2e allowance returns between January 2, 2008 and May 23, 2014. The approach is based on the hypothesis that the time path of the rate of CO2e returns is best described by a set of GARCH(p,q) processes. Moreover, we will test for the presence of asymmetries and leverage effects affecting returns and, finally, we will test for the presence of long memory in the conditional variance.

In all of the following, the conditional variance equations include a dummy variable "gap_t" which accounts for the number of non-trading days between day t and $t-1$ in order to capture the effect that volatility of returns increases after the weekends.

4.1 Conditional heteroskedasticity analysis

Table 2 presents the results on volatility in the CO2e allowance returns in the EU ETS market. Following the Box-Jenkins ARIMA modeling procedure and according to the Bayesian Information Criterion (BIC hereafter), the best model specification for the CO2e return rate is a simple AR(2) model without intercept and with no break dates. The first column of Table 2 presents the estimates of the coefficients of the mean equation, along with the residuals diagnosis. All the estimates are statistically significant at 1% significance. The Jarque-Bera (1980) test indicates that the residuals exhibit non-normality, whereby the distribution of residuals is leptokurtic and slightly right-skewed (these last results can be provided by the author upon request). The LM-test for

heteroskedasticity performed for 1, 20 and 100 lags is highly significant, which thereby strongly suggests the presence of an ARCH effect in returns. We confidently conclude that the $ARCH$ effect is present in the residuals and thus the variance is time-varying. The Breusch (1978)/Godfrey (1978) serial correlation LM test suggests that there is no evidence of serial correlation of any lag order up to 20, but we reject the null hypothesis of no serial correlation for lags greater than 20 for a 1% significance test. The Q_k portmanteau tests for the 20th, 50th and the 100th order serial correlation in $\hat{u}_t$ are high and lie within the rejection band, while for the 100th we cannot reject the null hypothesis of serial correlation. Finally, The Q_k^2 portmanteau tests based on $\hat{u}_t^2$ for the 20th, 50th and 100th order for the conditional mean are high, confirming that the square standardized residuals are uncorrelated for all models.

[Insert Table 2 around here]

4.2 Volatility and asymmetry in CO2e permit returns

Our analysis of volatility in CO2e allowance returns starts by estimating an ARMA(2,0-GARCH(1,1) model. The estimated coefficients and the corresponding residuals diagnosis for conditional variance are presented in the second column of Table 2. Given that the assumption of normality is violated, all the standard errors were computed using the robust Bollerslev-Wooldridge (1992) method. First note that the non-negativity condition is fulfilled as all coefficients are positive. The coefficients of both the *arch* and the *garch* terms are highly statistically significant. Furthermore, the sum of the lagged squared errors and the lagged projected variance is lower, but closer to unity. Accordingly, the effect of a shock on conditional variance will be temporary, although reverting to the mean at slower exponential rate. Moreover, the long-run daily volatility is nearly zero (0.000713).

The analyses of the presence of leverage effects are presented in both the third and fourth columns of Table 2. The AR(2), ω and δ parameters are not statistically significant in either model. However, the lagged squared residuals, the lagged GARCH and the leverage parameters are highly significant for a 1% test of significance. Moreover, the TGARCH performs better than the EGARCH (see both the Log-Likelihood and the two information criteria statistics values). For both models, the leveraged effect has the expected sign, thereby suggesting that bad news (namely bad weather conditions,

decrease in international energy prices, changes in allowances regime or special events such as international climate change negotiations and financial crisis) will increase volatility at a higher magnitude than good news on the same scale. In particular, considering the TGARCH model estimates, let us suppose that $\sigma_{t-1}^2 = 0.01$ and further consider that $\hat{u}_{t-1} = \pm 0.5$. If $\hat{u}_{t-1} = -0.5$; the fitted conditional variance would then be $\hat{\sigma}_t^2 = 0.045$, while if $\hat{u}_{t-1} = 0.5$, we should expect the fitted conditional variance for time t to be $\hat{\sigma}_t^2 = 0.023$. That is, a negative shock is expected to affect volatility by about 96% more than a positive shock of the same magnitude. Moreover, even though the magnitude effect is small (0.056) it is worth noting that, regarding the TGARCH model estimates, the leverage effect (0.087) is stronger (nearly 1.6 times) than the magnitude effect, suggesting that volatility is exceptionally affected by negative shocks, thereby increasing uncertainty during periods of turmoil. Moreover, it can also be seen that a negative shock of say, $u_{t-1}^2 = -0.5$ will have a greater impact on future volatility under TGARCH ($\sigma_t^2 = 0.037$) than under GARCH ($\sigma_t^2 = 0.027$), while a positive disturbance of the same extent will have more impact in GARCH (which is, obviously, $\sigma_t^2 = 0.027$) than in TGARCH ($\sigma_t^2 = 0.013$). This last result arises as a result of the reduction in the value of the parameter of the lagged squared residuals, α, when the asymmetry term is added into the model.

4.3 Long memory in CO2 returns volatility

From Table 2, it is clear that the standardized residuals and standardized squared residuals of the conditional mean are intertemporarily dependent, even for very long lags. In particular, the estimated value of $\alpha_1 + \beta_1$ is very close to unity, which indicates the possible existence of an integrated GARCH process. In order to further motivate the fractional integration empirical work, Figures 3 and 4 plot lag 3 through 1628 sample autocorrelations of the first difference and the fractionally different absolute returns, respectively, on the CO2e daily price from January 2, 2008 through April 23, 2014. From Figure 3 it is clear a that the absolute return correlations for very long lags frequently exceed the two 95% Bartlett (1946) confidence bands for no serial correlation. The Ljung-Box (1978) test is highly significant for any lag. For example, for lag 516, the Q statistic is 3,502.08 compared with the applicable value of the χ^2 distribution for 514 degrees of freedom, which is 567.

[Insert figure 3 around here]

In contrast, when we filter the original absolute returns series with a fractional differencing operator $(1-L)^{0.25}|y_t|$, the long-term dependence is substantially reduced, as shown in Figure 4. Indeed, the portmanteau Ljung-Box test for the joint significance for lag 516 is significantly reduced to 567.05, with a p-value of 0.056. Moreover, for all subsequent lags, the decision criterion suggests the absence of serial correlation.

[Insert figure 4 around here]

Table 3 shows the results of the IGARCH and FIGARCH models for the entire sample. The Ljung-Box (1978) Q_k portmanteau test for the kth-order serial correlation in $\hat{u}_t$ allows us to confidently reject the null hypothesis of uncorrelated returns. In addition, Table 3 also presents the Ljung-Box (1978) Q_k^2 portmanteau test based on $\hat{u}_t^2$ for homoskedasticity. Under the null hypothesis of conditional homoskedasticity, the statistic Q_k^2 will have a chi-squared distribution with k df. For both models, the null hypothesis is clearly rejected for k = 20, 50 and 100. These two results may be explained by the strong leptokurtosis and left skewness in returns and residuals, which affect the power of the Ljung-Box tests.

The estimated value of the dummy variable corresponding to the non-trading period δ is highly significant and has the expected sign in both models. Thus, the volatility of returns tends to increase after non-transaction periods, which is consistent with the empirical evidence for financial markets. However, this result contrasts with the estimated value in the previous GARCH models (and also in the conditional mean). This can be justified by the fact that the dummy only enters these models with a direct effect on the conditional variance equation.

The first column gives the estimated parameters of the general GARCH model with the constraint $\phi_1 = \alpha_1 + \beta_1 = 1$, (or IGARCH). This models the possibility of conditional variance being an $I(1)$ process, that is, an exogenous disturbance in conditional variance will permanently affect the predictions of conditional variance for all future periods. Except for the parameter $ar(2)$ of the conditional mean equation, all estimated parameters are significant at the 1% level. The GARCH parameter is high and,

unsurprisingly, very similar to the value obtained for the unrestricted model. However, both the AIC and BIC criteria choose the ARMA(2,0)-IGARCH(1,1) model over the ARMA(2,0)-GARCH(1,1) model.

Table 3 column 2 gives the results of the model estimation in which the possibility of volatility of returns is assumed to be the fractionally integrated process of order "d". The estimate of the fractional parameter d is between 0 and 1, thus allowing both the pure stationary (*d* = 0) and the unit root case (*d* = 1) to be rejected. More specifically, the estimated fractional parameter d is statistically significant at the 1% level, it lies within the interval (0.0 e 0.5) and is statistically different from these two bounds at a 1% level. The confidence interval for the estimated fractional integration parameter is relatively narrow and in the positive range. Also, the upper bound is lower than 0.5, thus indicating that volatility in returns is stationary and mean-reverting but exhibits long memory. In addition, the estimated parameters $\hat{\beta}_1 = 0.954$, $\hat{\phi}_1 = 0.997$ and $\hat{d} = 0.112$ satisfy the condition necessary for a non-negative conditional variance of the FIGARCH(1,*d*,1), models, namely $\beta_1 - d \leq \phi_1 \leq (2-d)/3$ and $d[\phi_1 - (1-d)/2] \leq \beta_1(\phi_1 - \beta_1 + d)$ [see Bollerslev and Mikkelsen (1996)]. Once again, the AIC and BIC criteria indicate that the ARMA(2,0)-FIGARCH(1,*d*,1) model is chosen over all others.

5. Conclusion

This paper models volatility in CO2e EUAs prices using an ARMA(2,0)-GARCH(p,q) class of models, using daily CO2e emissions allowances data from January 2, 2008 through May 23, 2014, for a total of T = 1,628 observations. Our results can be summarized as follows.

Firstly, we find that the conditional variance of daily EUA CO2e emissions allowance returns is time-varying, which is in line with recent findings of a relatively new research area called “carbon finance” [see, among others, Paolella and Taschini (2008) and Benz and Trück (2009), Chevallier (2009)].

Secondly, for the covariance-stationary GARCH model, all parameters are statistically significant at the standard significance levels, with the expected sign, and fulfill the non-

negative conditions. The conditional variance is highly persistent (0.896), and thereby the effects of a shock (whether positive or negative) will be temporary and short-lived.

Thirdly, we can confidently conclude that there is a leverage effect, so that volatility responds asymmetrically to positive and negative shocks. In particular, bad news will increase volatility at a higher magnitude than good news of the same magnitude. This result is important since it shows how the EU ETS market is vulnerable to the influence of exogenous factors such as atmospheric conditions, energy prices, emission allowance regimes, international negotiations on climate change, or financial crises.

Fourthly, for the fractionally integrated GARCH model the estimated fractional parameter is positive and smaller than 0.5. Therefore, we reject both the purely stationary case as well as the unit case, so that the conditional variance in CO2 emissions allowance is stationary and mean reverting, but with autocorrelations decaying at a hyperbolic rate. Therefore, in the FIGARCH model, a shock to the forecast of future conditional variance will be transitory but will last longer.

Our findings on long memory in the volatility of daily EUAs returns complement recent evidence by Conrad (2012) of an asymmetric power fractional differencing process in the conditional variance of intraday carbon returns, as well as by Liu and Weng (2013) on the fractional differencing process in the conditional variance of daily future carbon returns. In addition, the high degree of significance of the estimated fractional integration parameter in the conditional variance of CO2 emission allowances returns differs from the results of Feng et al. (2011), who did not find clear evidence of the presence of long memory. This discrepancy is certainly justified by the fact that our study benefits from a longer time series, which does not include the EU ETS pilot phase (2005-2007) but does include the first year of the third post-Kyoto negotiation period (2013-2020). Furthermore, it should be mentioned that after 2008, the institutional framework changed substantially, with the Commission playing a more influential role in national emission allowance plans, the creation of a single CAP for the EU, its extension to other ETS sectors, in particular aviation, and the introduction of new CO2 gases. In addition, in the third post-Kyoto negotiation period, traditional grandfathering-based allowanced allocation was replaced by an auction-based allocation process along with the creation of both a primary and a secondary CO2 emissions market, and a single EU-wide

electronic trade platform [the European Energy Exchange platform – EEX]. At the same time, a system of exceptions was also adopted for the auction scheme for sectors considered to be exposed to the risk of carbon leakage.

Our results have important implications for the decision-making process of the ETS industries. The presence of long memory suggests that returns already incorporate information about the relevant fundamentals to the formation of EUA prices. Thus knowledge about the stochastic properties of the conditional variance is of particular relevance for decisions to invest in abatement activities, for the design of arbitrage strategies to take advantage of momentary advantages in energy markets (oil, carbon and natural gas markets), for decisions on banking and borrowing, and for risk management in general. In addition, knowledge of the long-term presence of volatility in the EU ETS also allows the EU as well as Member State regulatory agencies to better design the regulatory framework for the functioning of this market.

The stochastic properties of the conditional variance of CO2 emission allowance returns are still relevant and should be taken into account when projections in CO2 emissions allowances are used to set prospective scenarios of public policies (either consumption or production based) or even private policies. This is particularly clear in the strategic orientation of some EU member-states in promoting green tax reforms that use the EUAs price as an index of the various instruments of these reforms [Pereira and Pereira (2013)].

Finally, our results also have important implications from a more technical perspective. Indeed, they suggest the importance of accounting for the interactions of volatility in the EUAs CO2 emissions market with energy sectors, the economy, and climate, both in terms of modeling and forecasting, as there is evidence that transitory shocks in conditional variance returns exhibit long memory. Indeed, given the strong connection of the energy and transport sectors to the rest of the economy, the effect of shocks may be transmitted to other sectors and even have impacts on the real economy, such as employment and output.

References

Aatola, P., M. Ollikainen and A. Toppinen (2013), “Price determination in the EU ETS market: Theory and econometric analisys with market fundamentals,” ***Energy Economics***, 36, 380-395;

Alberola, E., and J. Chevallier (2009); “ European Carbon Prices and Banking Restrictions: Evidence from Phase I (2005-2007),” ***The Energy Journal***, 30(3), 51-80.

Alberola, E., J. Chevallier and B. Cheze (2009); “ European carbon price fundamentals in 2005–2007: the effects of energy markets, temperatures and sectorial production,” Journal of Policy Modeling, 31(3), 446–462.

Alberola, E., J. Chevallier and B. Cheze (2008); “Price drivers and structural breaks in European carbon prices 2005–2007,” ***Energy Policy***, 36(2), 787–797;

Baillie, R. (1996); “Long memory and fractional integration in econometrics,” ***Journal of Econometrics,*** 73, 5–59;

Baillie, R., T. Bollerslev and H. Mikkelson. (1996). “Feactionally integrated generalized autoregressive conditional herteroskedasticity,” Journal of Econometrics, 74, 3-30.

Baillie R. and T. Bollerslev (1989) “The message in daily exchange rates: A conditional-variance tale,” Journal of Business & Economic Statistics, 7,(3) 297-305.

Bartlett, M. (1946); “On the theoretical specification and sampling properties of autocorrelated time series,” ***Supplement to the Journal of the Royal Statistical Society*** 8, 27-41;

Benz, E. and S. Trück (2009); “Modeling the price dynamics of CO2 emission allowances,” ***Energy Economics***, 31(1), 4-15;

Blyth, W. and D. Bunn (2011); “Coevolution of policy, market and technical price risks in the EU ETS,” ***Energy Policy***, 39(8), 4578-4593;

Bollerslev, T. and H. Mikkelsen (1996); “Modeling and pricing long memory in stc«ock market volatility,” ***Journal of Econometric***s, 73, 151-184.

Bollerslev, T. (1986); “Generalized autoregressive conditional heteroskedasticity,” ***Journal of Econometrics***, 31, 307-327;

Bollerslev, T. and J.M. Wooldridge (1992); “Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances,“***Econometric Reviews***, 11 (2), 143-172;

Bollerslev, T., R. Chou and K. Kroner, (1992), “ARCH modeling in finance: a review of the theory and empirical evidence,” Journal of Econometrics, 52, 5-59;

Breusch, T. (1978); "Testing for Autocorrelation in Dynamic Linear Models," ***Australian Economic Papers***, 17, 334–355.

Box, G. and D. Pierce, (1970), “Distribution of residual correlations in autoregressive-integrated moving average time series models,” ***Journal of the American Statistical Association***, 65, 1509–1526. doi: 10.2307/2284333.

Box, G. and G. Tiao (1975), “Intervention Analysis With Applications to Economic Environmental Problems,” ***Journal of the American Statistical Association***, 70, 70-79;

Chevallier, J. (2011a); “Detectiong instability in volatility of carbon prices,” ***Energy Economics***, 3(1);99-110;

Chevallier, J. (2011b); “Carbon price drivers: An updated literature review,” ***Halshs-00586513***, Hal-archives-ouvertes;

Chevallier, J. (2011c); A model of carbon interactions with macroeconomic and energy dynamics," Energy Economics, 33(6), 1295-1213.

Chevallier, J. (2009); "Carbon futures and macroeconomic risk factors: A view from the EU ETS," ***Energy Economics***, 31(4), 614-625.

Chevallier, J. and , E. Alberola (2009); "Banking and Borrowing in the EU ETS: An Econometric Appraisal of the 2005-2007," ***Intertemporal Market***;

Clò, S. (2010; "Grandfathering, auctioning and carbon leakage: assessing the inconsistencies of the new ETS Directive," Energy Policy, 38, 2420-2430;

Convery, F., and L. Redmond (2007); " Market and Price Developments in the European Union Emissions Trading Scheme," ***Review of Environmental Economics and Policy***, 1(1), 88-111.

Convery, F. (2009); "Reflections – The Emerging Literature on Emissions Trading in Europe," Review of Environmental Economics and Policy, 3(1), 121-137.

Christiansen, A. , A. Arvanitakis, K. Tangen and H. Hasselknippe (2005); "Price determinants in the EU emissions trading scheme," ***Climate Policy***, 5(1), 15–30.

Conrad, C., D. Ritter and W. Rotfub (2012); "Modeling and explaining the dynamics of European Union Allowance prices at high-frequency," ***Energy Economics***, 34(1), 316-326;

European Commission (2014); "Determining a list of sectors and subsectors which are deemed to be exposed to a significant risk of carbon leakage," ***Commission decision 2014/746/EU*** Available at:
http://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX%3A32014D0746

European Commission (2010); "Determining a list of sectors and subsectors which are deemed to be exposed to a significant risk of carbon leakage," ***Commission decision 2010/2/EU*** Available at:
http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex:32010D0002

Ellerman, A., F. Convery, and C. De Perthuis (2010), "Pricing Carbon: The European Union Emissions Trading Scheme," ***Cambridge University Press***, Cambridge, UK.

Ellerman A. and B. Buchner (2008); "Over-allocation or abatement? A preliminary analysis of the EU ETS based on the emissions," ***Environmental and Resource Economics***, 41, 267–287.

Ellerman, A. and B. Buchner (2007); "The European Union Emissions Trading Scheme: Origins, Allocation, and Early Results," ***Review of Environmental Economics and Policy***, 1(1), 66-87.

Engle, R., F. Lilien and D. Robin (1987); "Estimating time varying risk premium in the term structure: the ARCH-M model," ***Econometrica***, 55 (2), 391-407;

Engle R. and T. Bollerslev (1986) "Modelling the persistence of conditional variance," ***Econometric Reviews***, 5, 1-50;

Engle, R. (1982), "Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation, Econometrica 50, 987-1008;

Feng, Z-Hua, L. Zou and Y. Wei (2011); "Carbon price volatility: Evidence from EU ETS,"

French, K. and R. Roll (1986) "Stock return variances: The trivial of information and the reaction of traders," Journal of Financial Economics, 17, 5-26.

Godfrey, L. (1978); "Testing Against General Autoregressive and Moving Average Error Models when the Regressors Include Lagged Dependent Variables," ***Econometrica***, 46, 1293–1301;

Glosten, L., R. Jagannathan and D. Runkle (1993). "On the relation between the expected value and the volatility of the nominal excess return on a stocks," The Journal of Finance, 48 (5), 1779-1801.

Granger, C. (1980); "Long memory relationships and the aggregation of dynamic models," ***Journal of Econometrics*** 14, 227–38;

Granger C. and R. Joyeux (1980); "An introduction to long memory time series and fractional differencing," ***Journal of Times Series Analysis***, 1, 15–29

Helm, D. (2009); "EU climate-change policy: a critique," In Helm, D. ,Hepburn, C. (Eds.), "The Economics and Politics of Climate Change," ***Smith School Working Paper Series***, ***Oxford University Press***.

Hintermann, B. (2010); "Allowance price in the first phase of the EU ETS," ***Journal of Environmental Economic and Management,*** 59, 43-56.

Jarque, C. and A. Bera (1980); "Efficient tests for normality, homoscedasticity and serial independence of regression residuals," ***Economics Letters***, 6 (3): 255–259

Kettner, C., A. Koppl, S. Schleicher and G. Thenius (2007); "Stringency and distribution in the EU Emissions Trading Scheme—the 2005 evidence," ***FEEM - Nota di Lavoro*** 22;

Liu, H. and Y. Chen (2013); "A study on the volatility spillovers, long memory effects and interactions between carbon and energy markets: The impacts of extreme weather," ***Economic Modelling***, 35, 840-855;

Ljung, G. and P. Box (1978); "On a Measure of a Lack of Fit in Time Series Models," ***Biometrika,*** 65(2), 297–303;

Jarque, C. and A. Bera (1980). "Efficient tests for normality, homoscedasticity and serial independence of regression residuals." Economics Letters, pp. 6(3), pp. 255–259.

Mansanet-Bataller, M., J. Chevallier, M. Hervé-Mignucci and E. Alberola (2011); "EUA and sCER Phase II Price Drivers: Unveiling the reasons for the existence of the EUA-sCER spread," ***Energy Policy***, 39(3), 1056-1069;

Mansanet-Bataller M., A. Pardo and E. Valor (2007); "CO2 prices, energy and weather," ***The Energy Journal***, 28(3), 334–346.

Metcalf, G. (2009); "Cost Containment in Climate Change Policy: Alternative Approaches to Mitigating Price Volatility," ***University of Virginia Tax Review***, 29:2 (2009): 381 – 405.

Nelson, B. (1991); "Conditional heteroskedasticity in asset returns: A new approach," ***Econometrica***, 59, 347-370;

Neuhoff, K.,Martinez,K.,Sato,M.,2006. "Allocation, incentives and distortions: the impact of EUETS emissions allowance allocations to the electricity sector," ***Climate Policy***, 6, 73–91.

Oberndorfer U. (2009); "EU emission allowances and the stock market: evidence from the electricity industry," ***Ecological Economics***, 68(4), 1116–26.

Palma, W. (2007); "Long-memory times series: theory and methods," ***Wiley***, Hoboken;

Paolella, M., Taschini, L. (2008); "An econometric analysis of emission allowances prices." ***Journal of Banking and Finance*** 32, 2022–2032.

Parsons J., A. Ellerman and S. Feihauer (2009); "Designing a US market for CO2," ***Journal of Applied Corporate Finance***, 21(1), 79-86;

SEC 2008 {52}, (2008); Commission Staff Working Document Accompanying Document to the Proposal for a Directive of the European Parliament and of the Council Amending Directive 2003/87/EC so as to Improve and Extend the EU Greenhouse Gas Emission Allowance Trading System, Brussels, 23.1.2008, [http://eur-lex.europa.eu/legal-content/en/TXT/?uri=CELEX%3A52008SC0053].

Sowell F. (1992); "Modeling long-run behavior with the fractional ARIMA model," ***Journal of Monetary Economics,*** 29, 277–302.

Sowell F. (1992b); "Maximum likelihood estimation of stationary univariate fractionally integrated time series models," ***Journal of Econometrics***, 53, 165–88.

Tang, K. and W. Xiong (2012); "Index Investing and the Financialization of Commodities," ***Financial Analysts Journal***, 68 (6), 54-74.

Zakoïan J. (1994); "Threshold heteroskedastic model," ***Journal of Economic Dynamics and Control***, 18, 931-955;

Woerdman, E., A. Arcuri and S. Clò (2008); "Emissions trading and the polluter-pays principle: do polluters pay under grandfathering?" ***Review of Law and Economics***, 4 (2), 565-590.

Wood, P. and F. Jotzo (2011); "Price floors for emissions trading," ***Energy Policy***, 39(3), 1746-1753.

Zhang YJ, Y. Wei (2010); "An overview of current research on EU ETS: evidence from its operating mechanism and economic effect," ***Applied Energy,*** 87(6), 1804–14;

Zhen, F. L. Zou and Y. Wei (2011); "Carbon price volatility: evidence from EU ETS," ***Applied Energy***, 88, 590-598;

Table 1 - Summary statistics for daily CO2 prices and returns

Sub-periods	**m3**	**m4**	μ	σ_μ	$\left\|\sigma_\mu/\mu\right\|$
Prices (€)					
2008			22.27	0.226	0.010
2009			13.22	0.099	0.007
2010			14.37	0.064	0.004

2011			12.95	0.180	0.014
2012			7.37	0.045	0.006
2013			4.46	0.042	0.009
2014			5.58	0.075	0.013
Overall sample	0.593 (0.000)	-0.238 (0.056)	12.02	0.150	0.012
Returns					
2008			-0.115	0.145	1,259
2009			-0.041	0.195	4,717
2010			0.057	0.105	1,856
2011			-0.224	0.184	0,821
2012			0.013	0.187	14,843
2013			-0.001	0.294	498,393
2014			0.131	0.383	2,921
Overall sample	0.008 (0.902)	8.121 (0.000)	-0.041	0.078	1,913

Note: The statistics $m3$ and $m4$ are the standard measures of skewness and kurtosis. Under the null hypothesis of normality they will have asymptotic distributions of $m3 \approx N(0, 6/T)$ and $m4 \approx N(0, 24/T)$

Figure 1- Daily prices of CO2e emission Allowances in the ETS

Figure 2- Daily CO2e emission allowances returns in the ETS

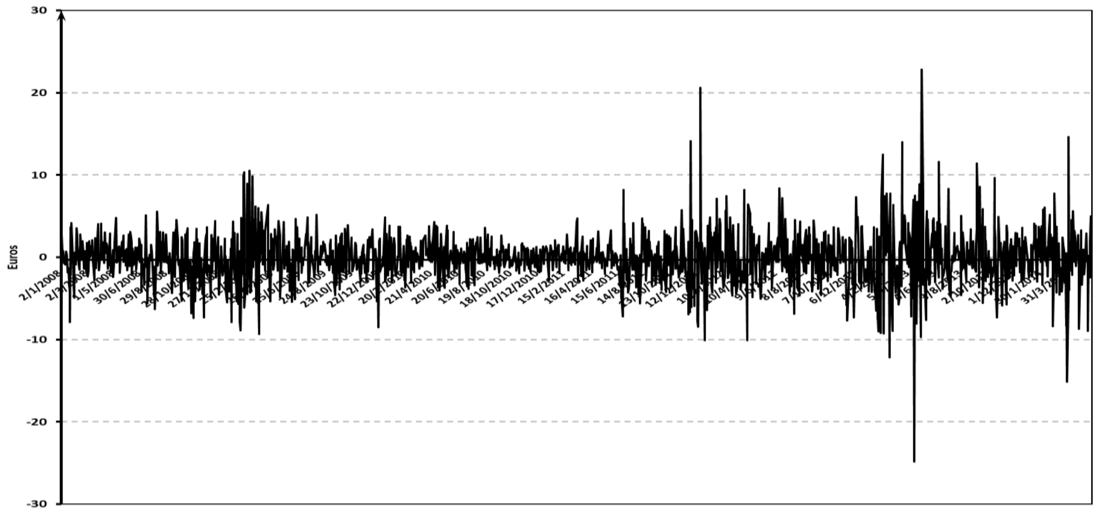


Table 2- ARMA(2,0)-GARCH(1,1) models for daily CO2 price returns

$y_t = \mu_1 y_{t-1} + \mu_2 y_{t-2} + u_t$

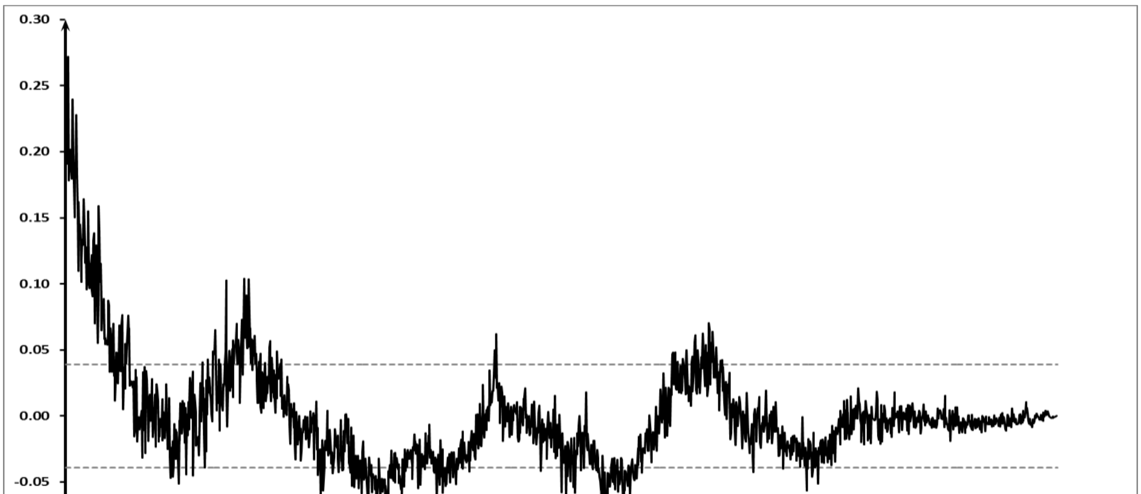
$h_t = \omega + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1} + \delta gap_t + \varepsilon_t, \quad z_t \equiv u_t\left(\sqrt{h_t}\right)^{-1}, E_{t-1}(z_t) = 0$ and $VAR_{t-1}(z_t) = 1$

Statistics	Mean (ARCH process)	GARCH	TGARCH	EGARCH
AR(1)	0.109 (0.000)	0.071 (0.023)	0.067 (0.023)	0.078 (0.049)
AR(2)	-0.099 (0.000)	-0.041 (0.192)	-0.037 (0.194)	-0.039 (0.161)
$\boldsymbol{\omega}$		7.13 e^{-8} (0.949)	4.21 e^{-7} (0.694)	-0.275 (0.008)
$\boldsymbol{\alpha_1}$		0.103 (0.000)	0.056 (0.006)	0.193 (0.000)
$\boldsymbol{\beta_1}$		0.896 (0.000)	0.897 (0.000)	0.982 (0.000)
$\boldsymbol{\gamma}$			0.087 (0.003)	-0.065 (0.001)
$\boldsymbol{\delta}$		1.40 e^{-6} (0.552)	1.11 e^{-6} (0.679)	-9.85 e^{-4} (0.886)
Log Likelihood	3340.418	3612.280	3626.180	3618.747

BIC	-4.097	-4.409	-4.424	-4.417
AIC	-4.104	-4.433	-4.447	-4.440
Jarque-Bera	3497.352 (0.000)			
LM-ARCH (1)	113.798 (0.000)			
LM-ARCH (20)	242.621 (0.000)			
LM-ARCH (100)	301.721 (0.000)			
LM-test (2)	1.052 (0.591)			
LM-test (20)	34.388 (0.024)			
LM-test (50)	79.811 (0.005)			
Q-stat (20)	34.638 (0.022)	48.676[1] (0.000)	49.003[2] (0.000)	47.120[3] (0.000)
Q-stat (50)	81.492 (0.003)	99.571 (0.000)	100.025 (0.000)	97.583 (0.000)
Q-stat (100)	122.82 (0.060)	142.707 (0.003)	143.085 (0.000)	140.752 (0.005)
Q^2-stat (20)	529.53 (0.000)	549.911 (0.000)	549.487 (0.000)	549.584 (0.000)
Q^2-stat (50)	679.44 (0.000)	698.593 (0.000)	698.160 (0.000)	698.323 (0.000)
Q^2-stat (100)	766.13 (0.000)	782.948 (0.000)	782.525 (0.000)	782.771 (0.000)

Note: p-values in brackets. The values of both the Box-Piearce and Ljung-Box portmanteau tests for up to kth-order serial correlation in the standardized residuals $\hat{\varepsilon}_t\hat{\sigma}_t^{-1}$, and the standardized squared residuals, $\hat{\varepsilon}_t^2\hat{\sigma}_t^{-1}$, are denoted by Q_k and Q_k^2 respectively.

Figure 3. Autocorrelations for absolute returns



Note: The 95% confidence bands for no serial dependence are also plotted in the figure

Figure 4. Autocorrelations for the fractionally differences ($d = 0.25$) of absolute returns

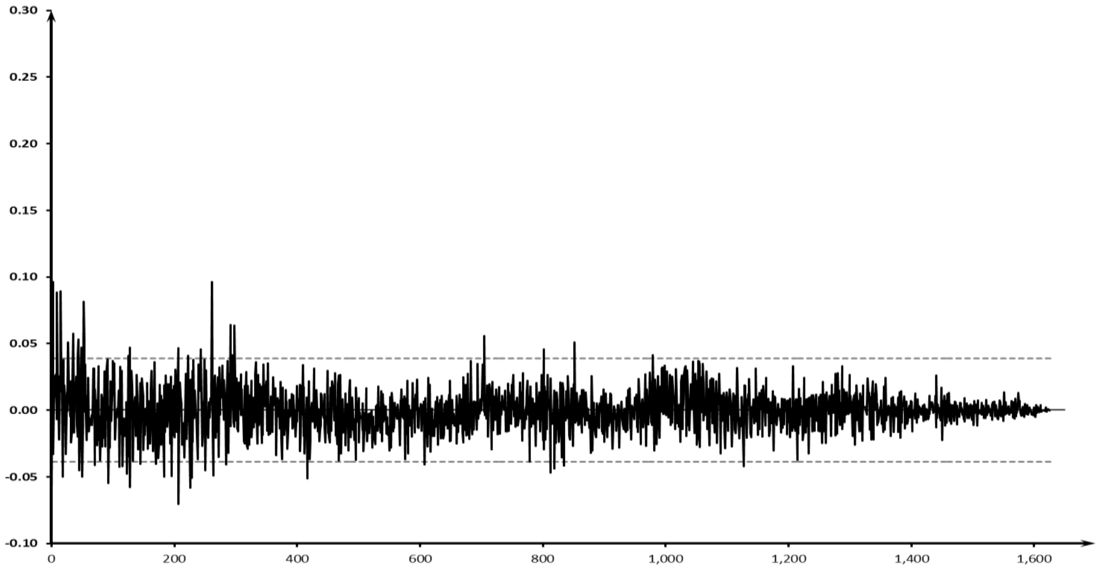


Table 3. AR(2)-IGARCH(1,0) and AR(2)-FIGARCH(1,0) models for daily CO2 price returns

$y_t = \mu_1 y_{t-1} + \mu_2 y_{t-2} + u_t$

$h_t = \omega(1-\beta_1)^{-1} + [1-(1-\beta_1 L)^{-1}(1-\phi_1 L)(1-L)^d][u_{t-1}^2 - \delta gap_t] + \delta gap_t$

$z_t \equiv u_t\left(\sqrt{h_t}\right)^{-1}$, $E_{t-1}(z_t) = 0$ and $VAR_{t-1}(z_t) = 1$

Statistics	ARMA(2,0)-IGARCH(1,1)	ARMA(2,0)-FIGARCH(1,d,1)
	All sample	All sample
AR(1)	0.066 (0.011)	0.060 (0.031)
AR(2)	-0.040 (0.114)	-0.043 (0.099)
$\boldsymbol{\omega}$	5.62 e^{-7} (0.004)	1.49 e^{-6} (0.260)
$\boldsymbol{\beta_1}$	0.905 (0.000)	0.954 (0.000)
$\boldsymbol{\phi_1}$	1.000 (0.000)	0.997 (0.000)
d		0.112 (0.001)

δ	5.79 e^{-6} (0.004)	5.98 e^{-5} (0.001)
Log Likelihood	3617.716	3621.791
BIC	-4.420	-4.420
Q-stat (20)	1044.737 (0.000)	1128.081 (0.000)
Q-stat (50)	1747.888 (0.000)	1889.810 (0.000)
Q-stat (100)	2379.882 (0.000)	2585.229 (0.000)
Q^2-stat (20)	205.018 (0.000)	417.694 (0.000)
Q^2-stat (50)	338.428 (0.000)	760.845 (0.000)
Q^2-stat (100)	463.707 (0.000)	1096.163 (0.000)
m3	-4.4909	-5.0149
m4	35.6806	46.2528

NOTE: p-values in brackets. The values of the Ljung-Box portmanteau test for up to kth-order serial correlation in the standardized residuals $\hat{\varepsilon}_t\hat{\sigma}_t^{-1}$, and the standardized squared residuals, $\hat{\varepsilon}_t^2\hat{\sigma}_t^{-1}$, are denoted by Q_k and Q_k^2 respectively.

The statistics $m3$ and $m4$ are the standard measures of skewness and kurtosis. Under the null hypothesis of normality they will have asymptotic distributions of $m3 \approx N(0, 6/T)$ and $m4 \approx N(0, 24/T)$