CEFAGE-UE Working Paper
2008/08

# Contagion effects of the US Subprime Crisis on Developed Countries

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**Abstract**

This study assesses whether capital markets of developed countries reflect the effects of financial contagion from the US subprime crisis and, in such case, if the intensity of contagion differs across countries. Adopting a definition of contagion that relates the phenomenon to an increase of cross-market linkages following a shock, copula models are used to analyse how the connections between the US and each market in the sample, evolved from the pre-crisis to the crisis period. The results suggest that markets in Canada, Japan, Italy, France and the United Kingdom display significant levels of contagion, which are less relevant in Germany. Canada appears to be the country where the highest intensity of contagion is observed.

**Key Words:** G7, subprime crisis, contagion, copula, event study

**JEL Classification:** F30, G14, G15

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## 1. INTRODUCTION

The burst of the US mortgage bubble, in August 2007, is pointed out as the moment when international financial markets were stroked by the subprime crisis (see, for instance, Toussaint, 2008). Notwithstanding the almost generalised interventions by central banks, suggesting that the impact could be global, until then the effects of the crisis were somewhat confined to the US.

After the first liquidity injection by the European Central Bank, taking place on the 9th of August, the supply of funds by central banks became almost a rule. By providing low cost money, monetary authorities wanted to ensure that commercial banks could maintain a normal level of activity, in spite of the increasing difficulties faced in the interbank money market. In fact, commercial banks were lending each other less frequently and at higher costs, either following an anticipation of losses and the consequent need to maintain adequate levels of reserves, or reflecting the turmoil in the financial system, motivated by the uncertainties on the real dimension of the crisis.

The latter was to some extent supported by the president of the Federal Reserve, who, in a speech delivered on the 15th of October, stated that the developments of the relatively small US subprime market were having a large impact upon the global financial system. In fact, losses associated with the subprime crisis have been incurred by institutions all over the developed world, including the G7 countries. The Citigroup, in the US, the Crédit Agricole in France, the HSBC in the United Kingdom, the CIBC in Canada, or the Deutsche Bank in Germany, are examples of banks reporting large losses associated with the subprime crisis. Following this, in the 9th of February 2008, members of the G7 met in Tokyo to discuss joint crisis control measures.

These episodes suggest that the burst of the US mortgage bubble is, in fact, affecting developed markets. In previous crisis, contagion effects were visible in stock market indices, and empirical assessments of financial contagion often focus on the dependence amongst stock market indices in turbulent periods (Bae et al., 2003). Cappiello et al. (2005), for instance, suggests that the financial crises occurred in the 90s, in Asia and Russia, affected Latin American markets. Rodriguez (2007) finds evidence of contagion in Asian markets during the 1997 Asian crisis.

In this study, an assessment of financial contagion effects from the US subprime crisis in G7 stock markets is performed. The adopted concept of contagion is proposed by Forbes and Rigobon (2001), according to which, financial contagion is ‘a significant increase in cross-market linkages after a shock to one country (or group of countries)’.[1] Following this, a significant increase in the dependence between the US market (the so-called *ground-zero market*) and the other markets in the analysed sample, from the pre-crisis period (i.e. before the bursting of the subprime mortgage bubble) to the crisis period (after the burst of the bubble), may be interpreted as evidence of contagion. When contagion exists, its intensity across markets is also evaluated. Apart form the G7 markets, the study also comprises the Portuguese’s, in order to appraise contagion effects in peripheral markets.

The concept of contagion proposed by Forbes and Rigobon, although not consensual, presents a number of operational advantages, highlighted by the authors.[2] Firstly, because it concentrates on the changes of relationships between markets, rather than on the magnitude of those relationships, it allows the assessment of the effectiveness of international diversification in periods of financial turmoil. A strategy of international diversification, as a means to decrease the risk of a portfolio without compromising its expected return, may be successful only if correlations between markets do not increase in times of crisis. It is therefore the change suffered by correlations, and not the correlations themselves that are of critical importance in such a context.

Secondly, it assesses the circumstances when an intervention of international authorities, in case of financial crisis, may be justified on the grounds of effectiveness. In fact, such type of bailing out intervention would only be adequate if the crisis entails a shortage of funds to a country presenting sound economic and policy fundamentals and no real links with the focus of the crisis. Following this, the concerted interventions by central banks as of August 2007 may be sub-optimal if the results of tests on contagion are negative in the context of the current subprime crisis.

Finally, the proposed definition of contagion avoids the difficult direct assessments of propagation mechanisms after a shock, by simply distinguishing between transmission mechanisms that change after a crisis and those that are a

[1] Forbes and Rigobon, 2001, p. 44.

2 See Forbes and Rigobon, 2001, pp. 45-46.

continuation of what previously existed. Furthermore, the proposed concept of contagion allows a classification of the theories that try to explain the mechanisms of international transmission as *non-crisis-contingent theories* (based on economic fundamentals and consistent with the absence of financial contagion) and *crisis-contingent theories* (based on investors' behaviour and expectations, and consistent with the existence of financial contagion). The work developed, *inter alia* by Mullainathan (2002), Valdés (1997), Calvo and Mendoza (2000) or Boyer et al. (2006) is consistent with the latter.

The remainder of this study is organised as follows: section 2 briefly surveys the relevant literature on copulas theory; section 3 presents the data and methodology, section 4 displays the empirical analysis and respective results, section 5 concludes.

## 2. COPULA THEORY

In spite of being relatively new in the context of empirical financial analyses, copulas are already an object of frequent use by researchers.[3] The copula concept was introduced by Sklar (1959) and may be used in finance as an alternative to correlations and other measures of relationships between variables, requiring rather strong assumptions, rarely met by financial variables. A copula is a joint distribution function of random variables, with pre-specified properties (see, for instance, Schmidt, 2006).

According to Sklar (1959), it is possible to split the joint distribution function in two basic components: the marginal variables function, following a uniform distribution in the interval [0, 1], and the function of dependence between these variables (i.e. the copula).[4] One important tool for the Sklar theorem is the fundamental result from the theory of generation of random numbers, by Fisher (1932), which states that if $X$ is a random continuous variable with a distribution function $F$, then $U = F(X)$ follows a uniform distribution between 0 and 1, regardless of the shape assumed by $F$. Variable $U$ is known in the literature as the probability integral transformation of $X$ (Patton,

[3] For its relevance in the issue at hand, see, inter alia, Embrechts et al. (2002) and Cherubini et al. (2004).

[4] In this study, bivariate continuous copulas are used, as the focus of the analysis is the structure of dependence between pairs of markets. The analysed copulas have, therefore, domain in the unitary square and contradomain in the unitary interval: $[0,1]\times[0,1]\rightarrow[0,1]$.

2002). In other words, a copula is a function allowing the link of univariate distribution functions to a joint distribution function. It was this capacity of expressing a link that inspired Sklar when he denominated such function as a copula – a word of Latin origin that means connection or junction (Patton, 2002).

Formally, the Sklar theorem states that any d-dimensional distribution function $F$, with univariate marginal distribution functions $F_1,...,F_d$, may be written in the following way:

$F(x_1,...,x_d) = C(F_1(x_1),...,F_d(x_d))$, where $C$ represents the copula. (1)

In alternative, if $X = (X_1,...,X_d)$ is a vector of random variables, the copula function is given by $C(u_1,...,u_d) = F(F_1^{-1}(u_1),...,F_d^{-1}(u_d))$, where $F_i^{-1}$ represents the marginal distribution inverse function $i$, with $U_i \sim Unif(0,1)$ (see the development in Nelsen, 2006).

Deriving both sides of equation (1), in order to each marginal variable, to obtain the density functions (here represented in lower case letters), the copula's role as a dependence structure is eventually more obvious:

$$\frac{\partial^d F(x_1,...,x_d)}{\partial x_1 ... \partial x_d} = \frac{\partial^d C(F_1(x_1),...,F_d(x_d))}{\partial F_1(x_1) ... \partial F_d(x_d)} \times \frac{\partial F_1(x_1)}{\partial x_1} \times ... \times \frac{\partial F_d(x_d)}{\partial x_d}$$

or

$$f(x_1,...,x_d) = c(u_1,...,u_d) \times f_1(x_1) \times ... \times f_d(x_d)$$

The above equation shows that, when the copula density function is neutral, the joint density function is equal to the product of the marginal density functions. In this case, all variables in vector $X = (X_1,...,X_d)$ are independent. If the copula density function is not neutral, it represents a dependence link amongst the variables in $X$.

An important feature of the Sklar theorem is the flexibility in multidimensional modelling. For instance, knowing the marginal distribution functions (which do not have to be identical) and knowing the copula function (that may be chosen independently of the marginal distributions), the joint distribution function may be directly obtained via the application of the theorem.

In this study, since the goal is the modelling of the dependence structure of pairs of financial series, by choosing the univariate distribution functions that are adequate for the marginals, and choosing an adequate copula to link these variables, it is possible to understand the co-movements of the series, using the points resulting from the integral transformation of the marginal variables probability as inputs to estimate the copula.

This means that it is possible to safely discard the Gaussian modelisation which, as shown in the literature, entails limitations when applied to some financial data, as a result of non-standard characteristics, such as heavy tails or stochastic volatility (ARCH effects). Furthermore, in the context of bivariate models, several studies have suggested that there are cases where the Gaussian distribution may not be appropriate, for it does not capture the asymmetric dependence often present in bidimensional series. Longin and Solnik (2001), Ang and Chen (2002), and Ang and Bakaert (2002), for instance, show that assets' returns appear to be more correlated in bearish than in bullish markets. Therefore, since the Gaussian distribution is symmetric, it is unsuitable when tails display some asymmetry.

A variety of copulas has been proposed (see for instance Nelsen, 2006) but, in finance, the most commonly adopted are the Gaussian copula (proposed by Lee (1983), the t-Student copula and some Archimedean copulas, such as those present in Gumbel (1960), Clayton (1978) or Frank (1979). When the variables of interest present a symmetric dependence structure, Gaussian or t-Student copulas may be adopted. If the dependence is more visible in the left hand side of the distribution, the Clayton's copula is more adequate, and the same for the Gumbel (1960) copula where variables display dependence in the right hand side of the band (Trivedi and Zimmer, 2005).

Although these two copulas cannot be used to model negative dependence structures between variables, this fact is not a problem when returns of stock indices are concerned, since dependence between them is usually positive. Frank's copula is symmetric but has some advantages in relation to the Gaussian and the t-Student copulas, namely to allow a more straightforward estimation of the dependence parameter, due to a simple analytical form. This copula is also appropriate to model variables displaying bands with weak dependence structures (Trivedi and Zimmer, 2005).

As an example, the functional forms of Clayton's and of Gumbel's copulas are displayed:

$$C^{Clayton}(u_1, u_2) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-\frac{1}{\theta}},$$

where $\theta \in (0,+\infty)$ represents the parameter of dependence between the marginal variables $X_1 = F_1^{-1}(U_1)$ and $X_2 = F_2^{-1}(U_2)$, being $F_1$ and $F_2$ the distribution functions of $X_1$ and $X_2$, respectively. As $\theta$ approaches 0, the variables become less dependent. Therefore, the bigger the $\theta$, the greater the dependence between $X_1$ and $X_2$.

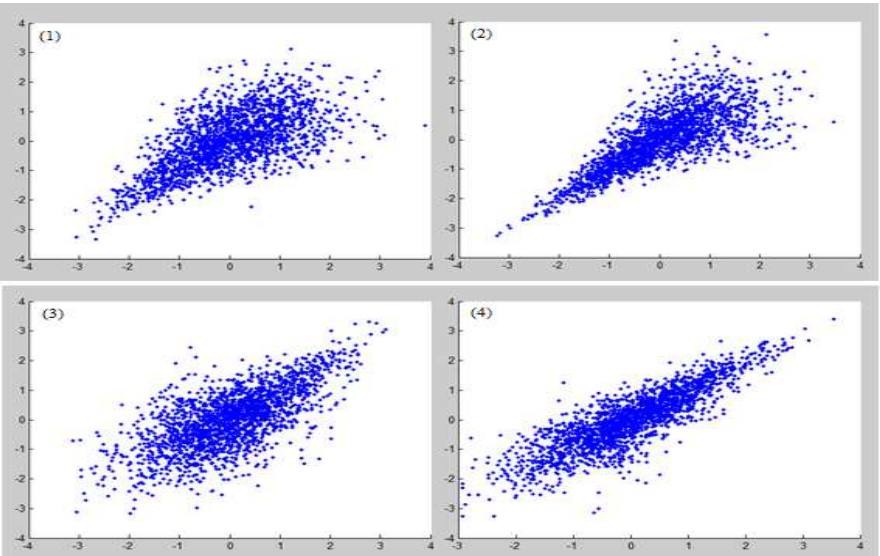
Gumbel's copula is represented by:

$$C^{Gumbel}(u_1,u_2) = \exp\left(-\left((-\ln u_1)^{\theta} + (-\ln u_2)^{\theta}\right)^{\frac{1}{\theta}}\right),$$

where the dependence parameter $\theta \in [1,+\infty)$. If $\theta = 1$, variables $X_1$ and $X_2$ are independent.[5] As $\theta$ increases, the dependence between the variables also increases.

The following figure shows a simulation of the behaviour of Clayton's and Gumbel's copulas for distinct dependence parameters. Standardised Gaussian distributions are assumed.

Figure 1. Random drawing of 2000 points departing from the copula of: (1) Clayton, with θ = 1.5; (2) Clayton, with θ = 3; (3) Gumbel, with θ = 2; (4) Gumbel, with θ = 3. It as been assumed for each panel that the marginal variables $X_1$ (in the horizontal axis) and $X_2$ (vertical axis) follow standardised Gaussian distributions




The Clayton's copula, in panel 2, displays a more concentrated distribution than that of panel 1, i.e. it exhibits a higher level of dependence. Furthermore, the left hand side of Clayton's copula is tighter than its right hand side – where the points are rather

[5] The independent copula is given by $C^{Indep}(u_1,u_2) = u_1 u_2$.

dispersed. These copulas may be adequate to model market indices exhibiting strong trends in down markets.

If the copula in panel 1 represents the dependence structure between two markets, in a period of calm, and the copula in panel 2 represents these markets' dependence structure in a period of crisis, the two copulas would convey evidence of financial contagion.

In addition to using 'pure' copulas, combinations of them may also be used (see, for instance Dias, 2004). The combination of a Grumbel and a Clayton copula, for instance, is adequate in the analysis of situations of almost perfect symmetry, but also for those of asymmetric shape.

The functional form of this mix copula is given by:

$$C^{mix}(u_1,u_2) = w_1 C^{Clayton}(u_1,u_2) + w_2 C^{Gumbel}(u_1,u_2),$$

where $w_1, w_2 \in [0,1]$ and $w_1 + w_2 = 1$

When $w_1$ tends to 1, the mixed copula tends to the Clayton copula and, as a consequence, the dependence in the left hand side of the mixed copula becomes more pronounced than that of the right hand side. Inversely, when $w_1$ tends to 0, the right hand side of the mixed copula becomes more prominent in terms of dependence. The mixed copula may also capture independence between variables, and it does so when the dependence parameter ($\theta$) of the Clayton copula is close to 0 and the Gumbel copula parameter is equal to 1.

## 3. DATA AND METHODOLOGY

The adopted methodology allows the comparison of dependence relationships in a period of relative financial stability, here referred to as the pre-crisis period, and in a turbulent phase, the crisis period. The pre-crisis period begins on the 1rst January 2005 and ends immediately before the burst of the subprime bubble, assumed to have occurred on the 1rst of August 2007. The crisis period starts at the beginning of August and extends until the 29th of February 2008, the last day for which data on stock market indices was collected.

Daily closing data on the Morgan Stanley Capital International (MSCI) indices denominated in the local currency are used for the G7 and the Portuguese markets.

The objective is the measurement of dependence between the US index and each of the remaining indices in the pre-crisis and in the crisis period. Thus, the following pairs of markets are assessed: US-Germany, US-Canada, US-France, US-Italy, US-Japan, US-Portugal and US-UK. The bivariate series are slightly different in length due to the need to calibrate the pairs of data according to each country's national holidays.[6]

Table 1. Number of daily observations for the bivariate series

| Sample lenght | USA/CAN | USA/FRA | USA/GER | USA/ITA | USA/JAP | USA/POR | USA/UK |
|---|---|---|---|---|---|---|---|
| Pre-crisis period | 635 | 641 | 641 | 639 | 615 | 641 | 637 |
| Crisis period | 144 | 146 | 144 | 143 | 137 | 146 | 145 |
| Total | 779 | 787 | 785 | 782 | 752 | 787 | 782 |

In the financial literature, the Pearson's linear correlation coefficient is one of the most used methods to quantify dependence (e.g., Bertero and Mayer, 1990; Baig and Goldfjan, 1999). However, authors such as Stambaugh (1995), Boyer, Gibson and Loretan (1999) or Forbes and Rigobon (2001), have shown that the use of this coefficient may produce weak results, if the analysed variables present conditional heteroskedasticity or autocorrelation, i.e. when the first and second moments of the distributions (mean and variance) are not stable over time. This is often not the case, as may be seen in figures 2 and 5, ahead. Furthermore, Corsetti, Pericoli and Sbracia (2005) have shown that when the variables are not independent and identically distributed (iid), the corrections made to the linear correlation coefficient, to accommodate the instability of the distributions' mean and variance, may still produce biased results. As Embrechts et al. (2003) and McNeil et al. (2005) have shown, the correlation coefficient is a robust measure of dependence only in the case of elliptic distributions, an example of which is the Gaussian distribution. If this is not the case, alternatives should be thought.

Following the problems associated with the use of simple correlations, other methods were adopted by researchers in the analysis of dependence between variables. Costinot et al. (2000), for instance, suggest the use of copulas, a framework that allows both an integral characterisation of the dependence link, but it also allows the definition

[6] Due to the different time zones, working hours in Japan and in the US do not overlap. Therefore, in order to ensure that the information contained in the US index is reflected in the Japanese index only in the next working day, the series of US data is lagged.

of its structure in scalar synthetic measures such as the *rank correlation*: the "Kendall's $\tau$ " or the "Spearman's $\rho$ " (Schmidt, 2006).

*Rank correlations* are also useful in the measurement of dependence structures between copulas. In fact, despite the fact that each copula has its own dependence parameter ($\theta$), they might not be easily comparable. As previously mentioned, the interval of variation of $\theta$ in a Clayton's copula, $(0,+\infty)$, differs from that of a Gumbel's copula, $(1,+\infty)$. The *rank correlation*, on the other hand, is comprised between -1 and 1, and is invariant to non linear transformations of the variables, as long as they are monotonic, as when a probability integral transformation is performed on the marginal variables.

In this analysis, and following the rational displayed so far, the Kendall's $\tau$ and the Spearman's $\rho$ are used as synthetic measures of dependence between the series of indices. These parameters are directly obtained from each copula's function, with the procedures described by Nelsen, (2006):

$$\rho_{Spearman}(X_1, X_2) = 12\int_0^1\int_0^1 (C(u_1,u_2) - u_1u_2)du_1du_2$$

$$\tau_{Kendall}(X_1, X_2) = 1 - 4\int_0^1\int_0^1 \frac{\partial C(u_1,u_2)}{\partial u_1}\frac{\partial C(u_1,u_2)}{\partial u_2}du_1du_2$$

Other than the *rank correlation*, asymptotic caudal coefficients, associated with the copulas ($\lambda_U$ and $\lambda_L$), are also often used as measures of dependence. These coefficients assess the probability of a random variable reaching an extreme value, in case other variable has already attained it. For instance, when an index has fallen sharply, it is possible to assess the probability of a similar behaviour on the part of other index by using the inferior asymptotic caudal coefficient ($\lambda_L$), formally defined as (see, for instanced, Schmidt, 2006):

$$\lambda_L = \lim_{q\to 0} P(X_2 \le F_2^{-1}(q) | X_1 \le F_1^{-1}(q))$$

Following this, the higher caudal coefficient is defined as:

$$\lambda_U = \lim_{q\to 1} P(X_2 \succ F_2^{-1}(q) \succ F_1^{-1}(q))$$

The methodological procedures adopted in this analysis may be divided into four distinct steps, described as follows:

**Step 1:** Elimination of the series' autoregressive and conditional heteroskedastic effects, with ARMA-GARCH models. The resulting residuals, denominated as filtered returns, are assessed for mean and variance stability.

**Step 2:** The samples of filtered returns are divided into two periods, the pre-crisis and the crisis period. Assuming that they are iid, a number of distribution functions for each index, and for the two periods, are estimated by maximum likelihood: the Gaussian, the t-location/scale, the logistic and the Gumbel's (extreme values). This last distribution is useful in the assessment of asymmetry in the sample of returns. The Akaike information criterion (AIC) is used to select the most adequate distribution.

**Step 3:** The distributions obtained in step 2 are used as inputs in the estimation, by maximum likelihood, of the various copulas, for each market and for the two periods. Again, the AIC is used to assess the quality of the estimates and in the selection of the most adequate copulas (see, for example, Dias, 2004).

This method of estimating the copulas' parameters is designed by McLeish and Small (1998) as the *Inference Functions for Margins* (IFM) and is developed in two parts. Firstly, the marginal distributions' parameters are estimated (as in step 2 above) and then used in the estimation of the copulas' parameters. One advantage of such procedure is the possibility of a priori testing the goodness of fit of the marginal distributions.

The 'pure' copulas tested are those of Clayton, Gumbel, Frank, Gaussian and the t-Student, and the mix copulas are those of Clayton-Gumbel, Gumbel-Survival Gumbel (see Dias, 2004) and Clayton-Gumbel-Frank.

**Step 4:** The bootstrap technique referred by Trivedi and Zimmer (2005) is used to calculate the variance-co-variance $V$ matrix for the estimated parameters and for the other remaining indicators associated with the copulas selected in step 3. The bootstrap procedure may be summarised as follows:

(1) Computation with the IFM method of the estimates for the vector of marginal distributions parameters ( $\hat{\beta}_1$ and $\hat{\beta}_2$ ), and for the vector of the copula ( $\hat{\theta}$ ). The vector of global estimated parameters is defined as $\hat{\Omega} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\theta})^T$ ;

(2) Definition of a sample of 'observations' obtained from the original data with a random draw with reposition;

(3) Use of this sample to re-estimate $\beta_1, \beta_2$ and $\theta$, using the IFM method;

(4) Replication of steps (2) and (3) $R$ times, being the $r$-th re-estimation identified by $\hat{\Omega}(r) = (\hat{\beta}_1(r), \hat{\beta}_2(r), \hat{\theta}(r))^T$ ;

(5) The parameters' standard deviations are the square roots of the main diagonal elements in matrix $V$ , estimated as follows: $\hat{V} = R^{-1} \sum_{r=1}^{R} (\hat{\Omega}(r) - \hat{\Omega})(\hat{\Omega}(r) - \hat{\Omega})^T$ .

The output of the bootstrap results is used in the assessment of the hypothesis of contagion. The test may be expressed as follows:

**Test 1 –** If contagion exists, the dependence or co-movement between markets is more intense during the period of crisis. Using the Kendall's $\tau$ :

$$\begin{cases} H_0 : \Delta\tau = \tau_{crisis} - \tau_{pre-crisis} \leq 0 \\ H_1 : \Delta\tau = \tau_{crisis} - \tau_{pre-crisis} > 0 \end{cases}$$

Using the Spearman's $\rho$ :

$$\begin{cases} H_0 : \Delta\rho = \rho_{crisis} - \rho_{pre-crisis} \leq 0 \\ H_1 : \Delta\rho = \rho_{crisis} - \rho_{pre-crisis} > 0 \end{cases}$$

Again using the bootstrap results, the hypothesis of differences in the contagion intensity between markets is tested as follows:

**Test 2 –** If contagion is more intense in market A than in market B, the increase in dependence between the US market and market A, from the pre-crisis to the crisis period, is higher than that between the US market and market B.

Using Kendall's $\tau$, the test may be expressed as:

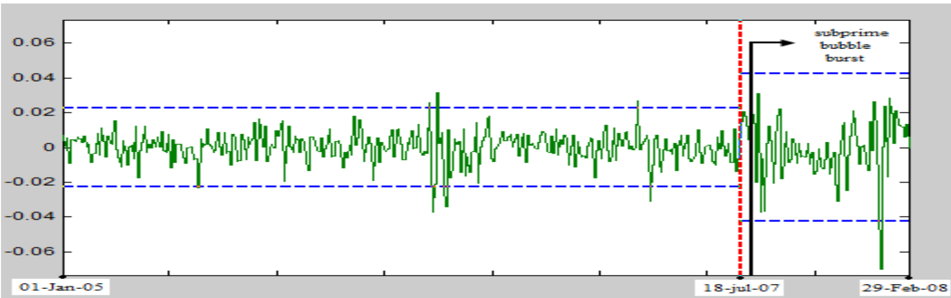
$$\begin{cases} H_0 : \Delta\tau_{A-B} = \left(\tau^{A}_{crisis} - \tau^{A}_{pre-crisis}\right) - \left(\tau^{B}_{crisis} - \tau^{B}_{pre-crisis}\right) \leq 0 \\ H_1 : \Delta\tau_{A-B} = \left(\tau^{A}_{crisis} - \tau^{A}_{pre-crisis}\right) - \left(\tau^{B}_{crisis} - \tau^{B}_{pre-crisis}\right) > 0 \end{cases}$$

## 4. RESULTS

**Step 1:** *Elimination of autoregressive effects and of conditional heteroskedasticity in the series*

In order to make sure that the first period is in fact a pre-crisis period, the series of returns of the different indices were decomposed to the scale 1, using a *wavelet* of Haar, as suggested by Misiti et al. (1997), and the main structure break occurred closed to the burst of the bubble was verified. The following figure displays the case of the French index, which structure break occurred on the 18th of July 2007.

Figure 2. Level 1 detail for the decomposition of the French index, developed using a Haar's wavelet




In order to eliminate trend dependence effects in the series, the procedures suggested inter alia by Dias (2004) and Gonzalo and Olmo (2005) are adopted. Firstly, through and analysis of the autocorrelation functions and of the Ljung-Box-Pierce and Engle's ARCH tests, the problems of temporal dependence are assessed (and the conclusion that these series are not iid is reached) either in means or in variances. Using the Box-Jenkins' method, an ARMA model is estimated for each index's average return.[7] GARCH (1,1) models were adjusted for the volatilities as they have been showed to be especially adequate for financial time series.

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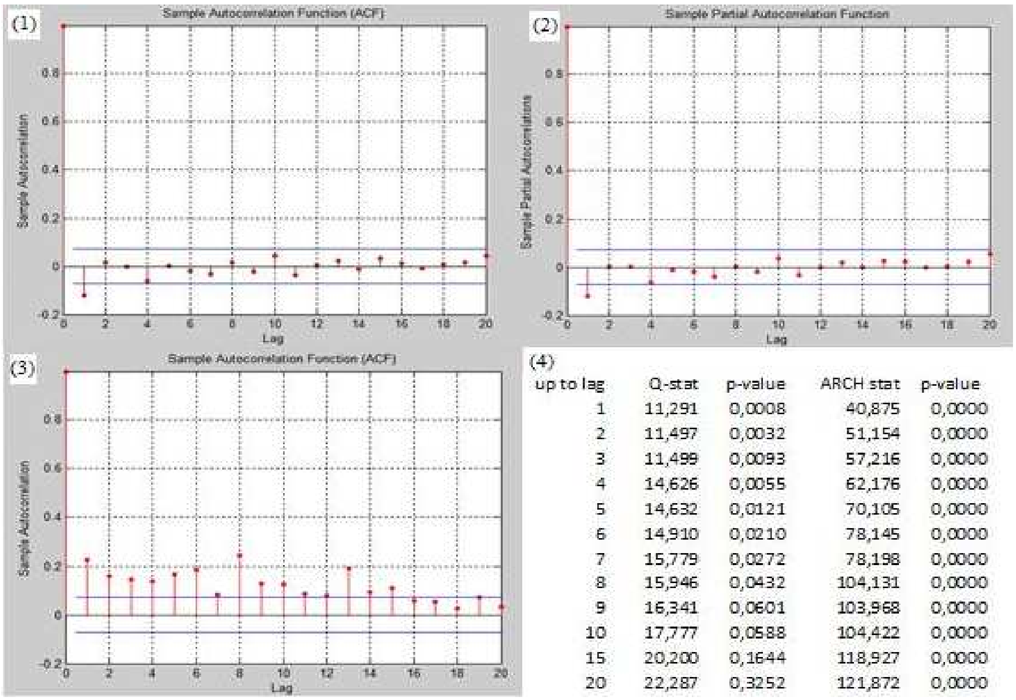
[7] An augmented Dickey-Fuller test is used to test for the absence of unit roots in the series and, therefore, to assess the adequacy of the proposed methods of analysis.

After estimating the ARMA-GARCH models, the filtered returns are recuperated. The tests previously described are performed to assess whether the identified problems are corrected. The three following figures highlight the process for the French market.

Figure 3. French Market. Returns' autocorrelation function (1), Square returns' autocorrelation function (3), returns' partial autocorrelation function (2), Q of Ljung-Box-Pierce and ARCH of Engle's tests (4)




| up to lag | Q-stat | p-value | ARCH stat | p-value |
|---|---|---|---|---|
| 1 | 11,291 | 0,0008 | 40,875 | 0,0000 |
| 2 | 11,497 | 0,0032 | 51,154 | 0,0000 |
| 3 | 11,499 | 0,0093 | 57,216 | 0,0000 |
| 4 | 14,626 | 0,0055 | 62,176 | 0,0000 |
| 5 | 14,632 | 0,0121 | 70,105 | 0,0000 |
| 6 | 14,910 | 0,0210 | 78,145 | 0,0000 |
| 7 | 15,779 | 0,0272 | 78,198 | 0,0000 |
| 8 | 15,946 | 0,0432 | 104,131 | 0,0000 |
| 9 | 16,341 | 0,0601 | 103,968 | 0,0000 |
| 10 | 17,777 | 0,0588 | 104,422 | 0,0000 |
| 15 | 20,200 | 0,1644 | 118,927 | 0,0000 |
| 20 | 22,287 | 0,3252 | 121,872 | 0,0000 |

The visual inspection of the figures suggests that AR and ARCH effects are present. Therefore, an ARMA(1,1)-GARCH(1,1) model was fitted. The obtained results are depicted in table 2.

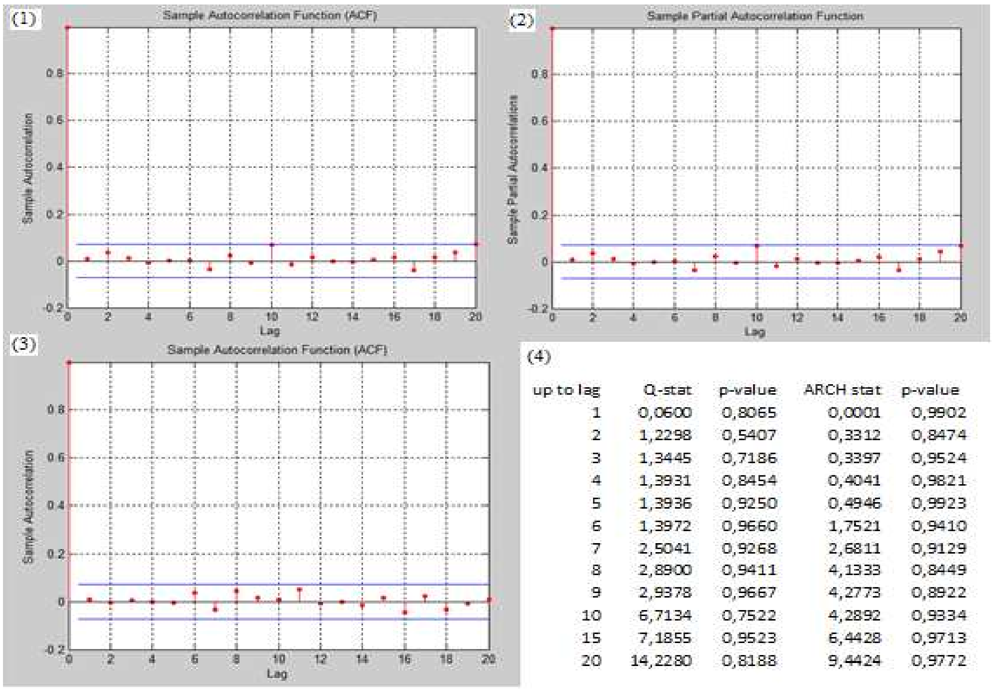
Table 2. French Market. ARMA(1,1)-GARCH(1,1) Model

| Parameter | Value | Std. Error | T statistic |
|---|---|---|---|
| C | 0.00026 | 0.00015 | 1.7156 |
| AR(1) | 0.66229 | 0.17493 | 3.7860 |
| MA(1) | -0.74397 | 0.15243 | -4.8806 |
| K | 4.0059e-6 | 1.2671e-6 | 3.1615 |
| GARCH(1) | 0.84468 | 0.03250 | 25.9883 |
| ARCH(1) | 0.11284 | 0.02237 | 5.0448 |

$C$ represents the constant associated with the mean's equation and $K$ is the constant associated with the variance's equation. It was assumed that the error term follows a Gaussian standard distribution.

Figure 4 contains the results of tests on the series of filtered returns.

Figure 4. French Market. Filtered returns' autocorrelation function (1), squared filtered returns' autocorrelation function (3), filtered returns' partial autocorrelation function (2), Ljung-Box-Pierce and ARCH of Engle tests (4)




| up to lag | Q-stat | p-value | ARCH stat | p-value |
|---|---|---|---|---|
| 1 | 0,0600 | 0,8065 | 0,0001 | 0,9902 |
| 2 | 1,2298 | 0,5407 | 0,3312 | 0,8474 |
| 3 | 1,3445 | 0,7186 | 0,3397 | 0,9524 |
| 4 | 1,3931 | 0,8454 | 0,4041 | 0,9821 |
| 5 | 1,3936 | 0,9250 | 0,4946 | 0,9923 |
| 6 | 1,3972 | 0,9660 | 1,7521 | 0,9410 |
| 7 | 2,5041 | 0,9268 | 2,6811 | 0,9129 |
| 8 | 2,8900 | 0,9411 | 4,1333 | 0,8449 |
| 9 | 2,9378 | 0,9667 | 4,2773 | 0,8922 |
| 10 | 6,7134 | 0,7522 | 4,2892 | 0,9334 |
| 15 | 7,1855 | 0,9523 | 6,4428 | 0,9713 |
| 20 | 14,2280 | 0,8188 | 9,4424 | 0,9772 |

The information contained in figure 4 suggests that time dependence problems in the French index are no longer relevant.

The results of the estimated models for the remaining indices are displayed in Table 3.[8]

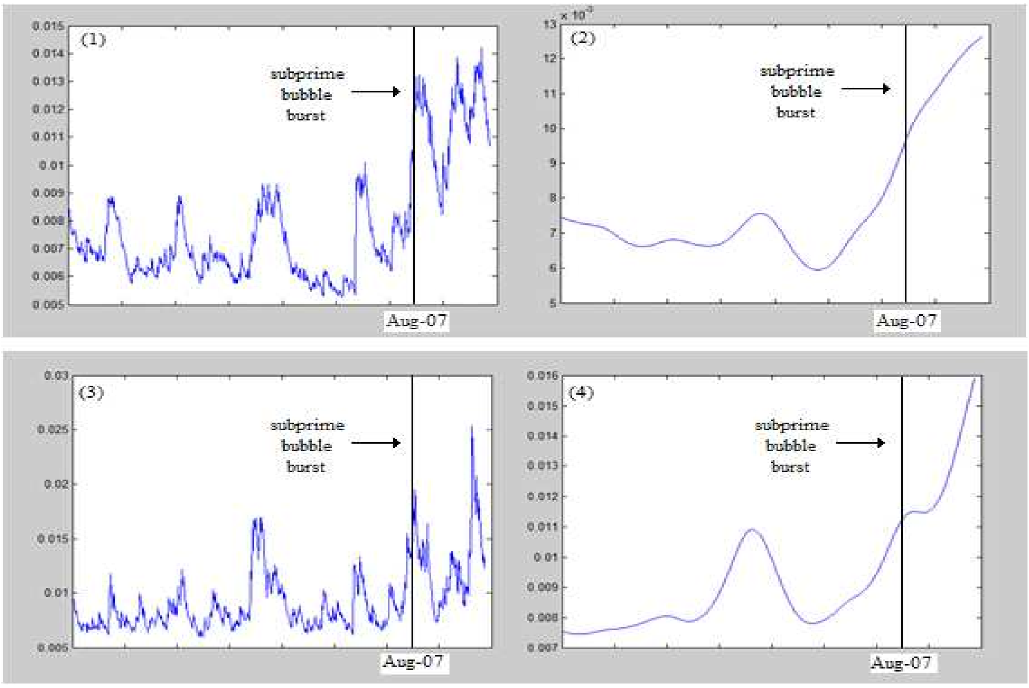
Table 3. Estimated models for the various indices

| Index | Model |
|---|---|
| Canada | ARMA(0,0)-GARCH(1,1) |
| France | ARMA(1,1)-GARCH(1,1) |
| Germany | ARMA(1,1)-GARCH(1,1) |
| Italy | ARMA(0,1)-GARCH(1,1), C=0 fixed |
| Japan | ARMA(0,0)-GARCH(1,1), C=0 fixed |
| Portugal | ARMA(0,0)-GARCH(1,1) |
| UK | ARMA(0,0)-GARCH(1,1) |
| USA | AR(1),AR(10),MA(1),MA(10)-GARCH(1,1), C=0 fixed |

In order to stress the distinct markets' behaviour in the pre-crisis and crisis periods, the series of conditional volatility for the US and French indices are displayed in Figure 5.

[8] As the dimension of the series of indices is variable (following the elimination of the holidays), and since the object of the assessment is the dependence towards the US, the size of each series was adjusted to that of the US. As a consequence, the ARMA-GARCH model for the US index undergoes small changes according to the dimension considered for the series. In Figure 5, the model corresponds to the US index associated to the US-France pair.

Figure 5. Panel (1): US index's conditional variance. Panel (2): US index' conditional variance after the application of the Hodrick Prescott filter (with a smoothing parameter of 1000000). Panel (3) French index's conditional variance. Panel (4): French index' conditional variance after the application of the Hodrick Prescott filter (parameter of 1000000)




The series' conditional volatility increases in the period after the burst of the bubble, thus depicting the turbulent environment in international markets.

**Step 2:** *Adjustment of the parametric distribution functions for the univariate series*

Table 4 contains information on the selected distribution function for each market.

Table 4. Distribution functions selected for the univariate series of the filtered returns

For the USA we used 641 observations for the pre-crisis period and 146 for the crisis period. The mean of the Logistic function has the same value as the location parameter; the variance is given by $\pi^2/3\sigma^2$. If X follows a t-location/scale distribution with $\nu>2$ degrees of freedom, then $(X-\mu)/\sigma$ follows a t-student distribution with mean 0 and variance given by $\nu/(\nu-2)$. The Extreme Value distribution has mean equals to $\mu+\gamma*\sigma$, where $\gamma$ is the Euler's constant; and variance given by $\pi*\sigma^2/6$. For the Gaussian distribution, the mean and variance are,respectively, $\mu$ and $\sigma^2$.

| Pre-crisis period | Selected distribution | Log Likelihood | AIC | μ - Location paramater (std. error) | σ - Scale paramater (std. error) | ν - Deg. freedom (std. error) |
|---|---|---|---|---|---|---|
| Canada | Logistic | 878.79 | -1753.58 | 0.0281<br>(0.0376) | 0.5439<br>(0.0179) | - |
| France | Logistic | 881.51 | -1759.01 | 0.0097<br>(0.0368) | 0.5370<br>(0.0177) | - |
| Germany | Logistic | 884.62 | -1765.24 | 0.0179<br>(0.0367) | 0.5373<br>(0.0178) | - |
| Italy | Logistic | 877.35 | -1750.70 | 0.0261<br>(0.0366) | 0.5348<br>(0.0177) | - |
| Japan | Logistic | 845.47 | -1686.94 | 0.0208<br>(0.0374) | 0.5352<br>(0.0181) | - |
| Portugal | t loc. - scale | 862.81 | -1719.63 | -0.0152<br>(0.0348) | 0.7716<br>(0.0358) | 5.5743<br>(1.1540) |
| UK | Logistic | 884.95 | -1765.90 | 0.0024<br>(0.0378) | 0.5469<br>(0.0180) | - |
| USA | Logistic | 871.67 | -1743.30 | 0.0622<br>(0.0359) | 0.5260<br>(0.0174) | - |
| **Crisis period** | **Selected distribution** | **Log Likelihood** | **AIC** | **μ - Location paramater (std. error)** | **σ - Scale paramater (std. error)** | **ν - Deg. freedom (std. error)** |
| Canada | Logistic | 219.57 | -435.13 | 0.0116<br>(0.0896) | 0.6208<br>(0.0434) | - |
| France | Gaussian | 221.28 | -438.56 | -0.2185<br>(0.0915) | 1.1053<br>(0.0650) | - |
| Germany | Logistic | 214.27 | -424.55 | -0.1460<br>(0.0865) | 0.5994<br>(0.0419) | - |
| Italy | Extreme value | 214.72 | -425.43 | 0.3344<br>(0.0847) | 0.9586<br>(0.0599) | - |
| Japan | t loc. - scale | 205.40 | -406.80 | -0.1400<br>(0.0928) | 0.9867<br>(0.0978) | 10.9886<br>(9.4228) |
| Portugal | Logistic | 215.20 | -426.39 | -0.2371<br>(0.0846) | 0.5908<br>(0.0410) | - |
| UK | Gaussian | 218.43 | -432.86 | -0.1090<br>(0.0910) | 1.0952<br>(0.0646) | - |
| USA | Gaussian | 224.24 | -444.48 | -0.0691<br>(0.0933) | 1.1279<br>(0.0663) | - |

The logistic distribution appears to be the more adequate alternative. The shape of the logistic distribution is quite similar to that of the t-Student, thus suggesting the existence of heavy tails in the filtered returns. Only the Italian market, during the crisis period, displays asymmetry in the distribution of the filtered returns. All remaining cases appear to be symmetric.

**Step 3:** *Adjustment of the copulas for the bivariate series in the pre-crisis and crisis periods*

Tables 5 and 6 display the copulas' estimates for the various markets in the two periods.

Table 5. Adjusted copulas for the pre-crisis period

| Copula models for Pre-crisis period | Dependence paramaters θ1 | θ2 | θ3 | Deg. of freedom | Weight parameters w1 | w2 | w3 | Log Likel. | AIC | BIC |
|---|---|---|---|---|---|---|---|---|---|---|
| **USA/CANADA** | | | | | | | | | | |
| Clayton | 0.9004 | - | - | - | - | - | - | 122.5 | -243.0 | -238.6 |
| Gumbel | 1.7099 | - | - | - | - | - | - | 148.1 | -294.3 | -289.8 |
| Frank | 4.3987 | - | - | - | - | - | - | 136.0 | -270.1 | -265.6 |
| Gaussian | 0.6277 | - | - | - | - | - | - | 158.1 | -314.3 | -309.8 |
| t-Student | 0.6262 | - | - | 29.2235 | - | - | - | 158.5 | -313.0 | -304.1 |
| Clayton-Gumbel | 1.1387 | 1.7625 | - | - | 0.2715 | 0.7285 | - | 152.8 | -299.7 | -286.3 |
| Gumbel-Survival Gumbel | 1.7461 | 1.6719 | - | - | 0.5998 | 0.4002 | - | 154.0 | -301.9 | -288.6 |
| Clayton-Gumbel-Frank | 1.1083 | 1.7500 | 22.9375 | - | 0.2711 | 0.7130 | 0.0159 | 152.9 | -295.8 | -273.5 |
| **USA/FRANCE** | | | | | | | | | | |
| Clayton | 0.5838 | - | - | - | - | - | - | 68.2 | -134.5 | -130.0 |
| Gumbel | 1.4382 | - | - | - | - | - | - | 77.6 | -153.1 | -148.7 |
| Frank | 2.8755 | - | - | - | - | - | - | 64.2 | -126.4 | -121.9 |
| Gaussian | 0.4672 | - | - | - | - | - | - | 78.8 | -155.7 | -151.2 |
| t-Student | 0.4525 | - | - | 6.2160 | - | - | - | 86.6 | -169.2 | -160.2 |
| Clayton-Gumbel | 0.6738 | 1.4662 | - | - | 0.3431 | 0.6569 | - | 84.4 | -162.7 | -149.3 |
| Gumbel-Survival Gumbel | 1.4675 | 1.3767 | - | - | 0.5394 | 0.4606 | - | 85.0 | -164.1 | -150.7 |
| Clayton-Gumbel-Frank | 0.7197 | 1.4688 | -99.9375 | - | 0.3393 | 0.6513 | 0.0094 | 85.9 | -161.9 | -139.6 |
| **USA/GERMANY** | | | | | | | | | | |
| Clayton | 0.5652 | - | - | - | - | - | - | 67.8 | -133.7 | -129.2 |
| Gumbel | 1.4354 | - | - | - | - | - | - | 75.8 | -149.5 | -145.1 |
| Frank | 2.8904 | - | - | - | - | - | - | 63.9 | -125.8 | -121.3 |
| Gaussian | 0.4599 | - | - | - | - | - | - | 76.5 | -151.0 | -146.5 |
| t-Student | 0.4488 | - | - | 6.3327 | - | - | - | 84.7 | -165.4 | -156.6 |
| Clayton-Gumbel | 0.4867 | 1.5977 | - | - | 0.4423 | 0.5577 | - | 83.8 | -161.6 | -148.2 |
| Gumbel-Survival Gumbel | 1.6731 | 1.2502 | - | - | 0.4716 | 0.5284 | - | 83.6 | -161.1 | -147.7 |
| Clayton-Gumbel-Frank | 0.5280 | 1.5883 | -100.0000 | - | 0.4345 | 0.5600 | 0.0055 | 84.6 | -159.2 | -136.9 |
| **USA/ITALY** | | | | | | | | | | |
| Clayton | 0.5487 | - | - | - | - | - | - | 66.8 | -131.6 | -127.2 |
| Gumbel | 1.4539 | - | - | - | - | - | - | 77.6 | -153.4 | -148.9 |
| Frank | 2.8616 | - | - | - | - | - | - | 63.9 | -125.7 | -121.2 |
| Gaussian | 0.4660 | - | - | - | - | - | - | 78.0 | -154.1 | -149.6 |
| t-Student | 0.4544 | - | - | 7.5412 | - | - | - | 82.9 | -161.8 | -152.9 |
| Clayton-Gumbel | 0.5081 | 1.5742 | - | - | 0.4008 | 0.5992 | - | 83.6 | -161.2 | -147.9 |
| Gumbel-Survival Gumbel | 1.6094 | 1.2783 | - | - | 0.5180 | 0.4820 | - | 83.8 | -161.6 | -148.2 |
| Clayton-Gumbel-Frank | 0.5649 | 1.5625 | -99.9375 | - | 0.3844 | 0.6070 | 0.0086 | 84.4 | -158.8 | -136.5 |
| **USA/JAPAN** | | | | | | | | | | |
| Clayton | 0.4563 | - | - | - | - | - | - | 49.5 | -97.0 | -92.6 |
| Gumbel | 1.3027 | - | - | - | - | - | - | 42.4 | -82.8 | -78.4 |
| Frank | 2.1449 | - | - | - | - | - | - | 35.8 | -69.6 | -65.1 |
| Gaussian | 0.3911 | - | - | - | - | - | - | 51.3 | -100.7 | -96.2 |
| t-Student | 0.3761 | - | - | 16.1039 | - | - | - | 52.3 | -100.6 | -91.7 |
| Clayton-Gumbel | 0.4700 | 1.3985 | - | - | 0.6277 | 0.3723 | - | 54.2 | -102.3 | -89.1 |
| Gumbel-Survival Gumbel | 1.3892 | 1.2581 | - | - | 0.2940 | 0.7060 | - | 52.9 | -99.8 | -86.5 |
| Clayton-Gumbel-Frank | 0.4700 | 1.3985 | 2.0420 | - | 0.6278 | 0.3722 | 0.0000 | 54.2 | -98.3 | -76.2 |
| **USA/PORTUGAL** | | | | | | | | | | |
| Clayton | 0.2580 | - | - | - | - | - | - | 17.6 | -33.2 | -28.7 |
| Gumbel | 1.1418 | - | - | - | - | - | - | 12.1 | -22.2 | -17.7 |
| Frank | 1.3132 | - | - | - | - | - | - | 14.7 | -27.4 | -23.0 |
| Gaussian | 0.2160 | - | - | - | - | - | - | 15.4 | -28.8 | -24.3 |
| t-Student | 0.2192 | - | - | 11.2065 | - | - | - | 18.8 | -33.6 | -24.7 |
| Clayton-Gumbel | 0.1952 | 2.1250 | - | - | 0.8880 | 0.112 | - | 19.7 | -33.4 | -20.1 |
| Gumbel-Survival Gumbel | 1.0938 | 1.1875 | - | - | 0.2866 | 0.7134 | - | 18.4 | -30.8 | -17.4 |
| Clayton-Gumbel-Frank | 0.1954 | 2.7656 | 1.7188 | - | 0.8025 | 0.0723 | 0.1252 | 19.8 | -19.8 | -7.3 |
| **USA/UK** | | | | | | | | | | |
| Clayton | 0.5354 | - | - | - | - | - | - | 58.3 | -114.5 | -110.1 |
| Gumbel | 1.4200 | - | - | - | - | - | - | 73.5 | -145.0 | -140.6 |
| Frank | 2.7234 | - | - | - | - | - | - | 58.9 | -115.8 | -111.3 |
| Gaussian | 0.4508 | - | - | - | - | - | - | 71.9 | -141.8 | -137.3 |
| t-Student | 0.4378 | - | - | 6.8288 | - | - | - | 78.0 | -152.1 | -143.2 |
| Clayton-Gumbel | 0.5840 | 1.4580 | - | - | 0.2988 | 0.7012 | - | 77.3 | -148.7 | -135.3 |
| Gumbel-Survival Gumbel | 1.4531 | 1.3438 | - | - | 0.6195 | 0.3805 | - | 78.0 | -150.0 | -136.6 |
| Clayton-Gumbel-Frank | 3.9141 | 1.4063 | -1.5000 | - | 0.0806 | 0.8793 | 0.0401 | 77.2 | -144.5 | -122.2 |

Table 6. Adjusted copulas for the crisis period

| Copula models for Crisis period | Dependence paramaters $\theta_1$ | $\theta_2$ | $\theta_3$ | Deg. of freedom | Weight parameters $w_1$ | $w_2$ | $w_3$ | Log Likel. | AIC | BIC |
|---|---|---|---|---|---|---|---|---|---|---|
| **USA/CANADA** | | | | | | | | | | |
| Clayton | 1.8387 | - | - | - | - | - | - | 64.0 | -126.1 | -123.1 |
| Gumbel | 2.3654 | - | - | - | - | - | - | 66.6 | -131.1 | -128.1 |
| Frank | 8.4777 | - | - | - | - | - | - | 72.5 | -143.0 | -140.0 |
| Gaussian | 0.7812 | - | - | - | - | - | - | 68.0 | -133.9 | -130.9 |
| t-Student | 0.8087 | - | - | 4.1043 | - | - | - | 76.8 | -149.6 | -143.6 |
| Clayton-Gumbel | 1.5615 | 3.2969 | - | - | 0.3919 | 0.6081 | - | 77.2 | -148.4 | -139.5 |
| Gumbel-Survival Gumbel | 3.7023 | 1.9999 | - | - | 0.4487 | 0.5513 | - | 77.6 | -149.1 | -140.2 |
| Clayton-Gumbel-Frank | 1.3027 | 3.0000 | 17.0000 | - | 0.3753 | 0.2322 | 0.3925 | 79.7 | -149.4 | -134.6 |
| **USA/FRANCE** | | | | | | | | | | |
| Clayton | 0.6077 | - | - | - | - | - | - | 16.1 | -30.2 | -27.2 |
| Gumbel | 1.5689 | - | - | - | - | - | - | 22.6 | -43.3 | -40.3 |
| Frank | 4.1226 | - | - | - | - | - | - | 26.1 | -50.2 | -47.3 |
| Gaussian | 0.5072 | - | - | - | - | - | - | 21.7 | -41.4 | -38.4 |
| t-Student | 0.5383 | - | - | 7.5719 | - | - | - | 23.6 | -43.3 | -37.3 |
| Clayton-Gumbel | 9.9193 | 1.4609 | - | - | 0.1346 | 0,8654 | - | 25.2 | -44.4 | -35.5 |
| Gumbel-Survival Gumbel | 1.4424 | 7.2802 | - | - | 0.8702 | 0.1298 | - | 25.0 | -44.0 | -35.1 |
| Clayton-Gumbel-Frank | 11.9210 | 1.0313 | 5.0000 | - | 0.0854 | 0.2060 | 0.7086 | 28.2 | -46.4 | -31.5 |
| **USA/GERMANY** | | | | | | | | | | |
| Clayton | 0.5683 | - | - | - | - | - | - | 14.2 | -26.4 | -23.4 |
| Gumbel | 1.4976 | - | - | - | - | - | - | 18.7 | -35.4 | -32.4 |
| Frank | 3.6899 | - | - | - | - | - | - | 21.8 | -41.7 | -38.7 |
| Gaussian | 0.4745 | - | - | - | - | - | - | 18.4 | -34.7 | -31.7 |
| t-Student | 0.5058 | - | - | 9.0420 | - | - | - | 20.5 | -37.1 | -31.2 |
| Clayton-Gumbel | 0.9944 | 1.5245 | - | - | 0.2945 | 0.7055 | - | 20.1 | -34.3 | -25.4 |
| Gumbel-Survival Gumbel | 1.5017 | 1.5414 | - | - | 0.6400 | 0.3600 | - | 19.9 | -33.8 | -24.9 |
| Clayton-Gumbel-Frank | 0.0005 | 2.3594 | 4.2266 | - | 0.1322 | 0.0763 | 0.7915 | 22.1 | -34.1 | -19.3 |
| **USA/ITALY** | | | | | | | | | | |
| Clayton | 0.6661 | - | - | - | - | - | - | 14.8 | -27.5 | -24.6 |
| Gumbel | 1.5494 | - | - | - | - | - | - | 22.1 | -42.2 | -39.2 |
| Frank | 4.4063 | - | - | - | - | - | - | 27.8 | -53.6 | -50.6 |
| Gaussian | 0.5408 | - | - | - | - | - | - | 23.7 | -45.5 | -42.5 |
| t-Student | 0.5507 | - | - | 22.7782 | - | - | - | 24.0 | -43.9 | -38.0 |
| Clayton-Gumbel | 4.3335 | 1.5216 | - | - | 0.1928 | 0.8072 | - | 24.9 | -43.8 | -34.9 |
| Gumbel-Survival Gumbel | 1.6269 | 1.6504 | - | - | 0.6455 | 0.3545 | - | 24.3 | -42.6 | -33.7 |
| Clayton-Gumbel-Frank | 9.5434 | 1.4063 | 4.5000 | - | 0.0578 | 0.1639 | 0.7783 | 28.3 | -46.6 | -31.8 |
| **USA/JAPAN** | | | | | | | | | | |
| Clayton | 0.5183 | - | - | - | - | - | - | 12.7 | -23.5 | -20.6 |
| Gumbel | 1.5574 | - | - | - | - | - | - | 23.7 | -45.4 | -42.5 |
| Frank | 3.4463 | - | - | - | - | - | - | 18.75 | -35.5 | -32.6 |
| Gaussian | 0.5169 | - | - | - | - | - | - | 21.3 | -40.6 | -37.7 |
| t-Student | 0.5182 | - | - | 17.7954 | - | - | - | 21.5 | -39.0 | -33.1 |
| Clayton-Gumbel | 0.9095 | 1.5574 | - | - | 0.0000 | 1.0000 | - | 23.7 | -41.4 | -32.6 |
| Gumbel-Survival Gumbel | 1.5574 | 1.6550 | - | - | 1.0000 | 0.0000 | - | 23.7 | -41.4 | -32.6 |
| Clayton-Gumbel-Frank | 0.9095 | 1.5574 | 2.8875 | - | 0.0000 | 1.0000 | 0.0000 | 23.7 | -37.4 | -22.8 |
| **USA/PORTUGAL** | | | | | | | | | | |
| Clayton | 0.2157 | - | - | - | - | - | - | 2.8 | -3.5 | -0.5 |
| Gumbel | 1.2161 | - | - | - | - | - | - | 5.3 | -8.5 | -5.5 |
| Frank | 1.9094 | - | - | - | - | - | - | 6.7 | -11.4 | -8.4 |
| Gaussian | 0.2701 | - | - | - | - | - | - | 5.5 | -9.1 | -6.1 |
| t-Student | 0.2701 | - | - | 4.6692 | - | - | - | 5.5 | -7.1 | -1.1 |
| Clayton-Gumbel | 0.0891 | 1.2813 | - | - | 0.2105 | 0.7895 | - | 5.2 | -4.4 | 4.5 |
| Gumbel-Survival Gumbel | 1.1563 | 2.7188 | - | - | 0.8787 | 0.1213 | - | 5.9 | -5.7 | 3.2 |
| Clayton-Gumbel-Frank | 4.8571 | 1.0302 | 1.6314 | - | 0.0635 | 0.0000 | 0.9365 | 7.0 | -4.0 | 10.9 |
| **USA/UK** | | | | | | | | | | |
| Clayton | 0.5720 | - | - | - | - | - | - | 15.9 | -29.8 | -26.8 |
| Gumbel | 1.5054 | - | - | - | - | - | - | 19.4 | -36.8 | -33.8 |
| Frank | 3.8021 | - | - | - | - | - | - | 23.0 | -44.0 | -41.1 |
| Gaussian | 0.5089 | - | - | - | - | - | - | 21.7 | -41.5 | -38.4 |
| t-Student | 0.5089 | - | - | high | - | - | - | 21.7 | -39.5 | -33.5 |
| Clayton-Gumbel | 0.4158 | 2.0000 | - | - | 0.4853 | 0.5147 | - | 21.7 | -37.4 | -28.5 |
| Gumbel-Survival Gumbel | 2.0469 | 1.2656 | - | - | 0.4645 | 0.5355 | - | 21.4 | -36.8 | -27.9 |
| Clayton-Gumbel-Frank | 50.0000 | 1.0005 | 3.6978 | - | 0.0402 | 0.0307 | 0.9291 | 25.3 | -40.7 | -25.8 |

A number of aspects are of interest:

- Firstly, the various estimated copulas' parameters increase from the pre-crisis to the crisis period, thus suggesting that the co-movements between markets became more pronounced after the burst of the mortgage bubble.

- Secondly, the level of dependence between each of the markets and the US is not homogeneous. Focusing on the results obtained with the t-Student copula, for the pre-crisis period, the Canadian market displays the highest level of dependence in relation to the US market, presenting a coefficient of 0,6262. The German, French, Italy and UK markets display lower levels of dependence, presenting values around 0,45. The least dependent markets are those of Japan (0,3761) and Portugal (0,2192). In spite of the displayed distinct levels of dependence, all markets present positive dependence coefficients, thus suggesting that they are all connected with the US and evolving in the same direction.

- Finally, the t-Student copula appears to be the more adequate to model the dependence structure between the markets in the pre-crisis period, whereas Frank's copula outperforms the others for the crisis period. The copulas selected with the AIC present a symmetric structure, in contrast with the results by Longin and Solnik (2001), Ang and Chen (2002), and Ang and Bakaert (2002), which suggest that the higher level of dependence between markets occurs in periods of decreasing returns. The exception is the Japanese market, exhibiting higher dependence in the right tail, in the crisis period, as the representative copula is the Gumbel's.

**Step 4:** *Computation of the parameters' standard deviations and of the statistics associated with the copulas. Assessing the hypotheses of contagion and of distinct contagion intensities*

Table 7. Selected Copulas

| | USA / CANADA | USA / FRANCE | USA / GERMANY | USA / ITALY | USA / JAPAN | USA / PORTUGAL | USA / UK |
|---|---|---|---|---|---|---|---|
| | | | | Pre-crisis period | | | |
| Selected copula | Gaussian | t-Student | t-Student | t-Student | Clay.-Gumb. | t-Student | t-Student |
| Depend. param. ($\theta_1$) | 0.6277 (0.0249) | 0.4525 (0.0373) | 0.4488 (0.0366) | 0.4544 (0.0350) | 0.4700 (0.0718) | 0.2192 (0.0395) | 0.4378 (0.0351) |
| Depend. param. ($\theta_2$) | - | - | - | - | 1.3985 (0.1844) | - | - |
| Weight param. ($w_1$) | - | - | - | - | 0.6277 (0.1333) | - | - |
| Weight param. ($w_2$) | - | - | - | - | 0.3723 (0.1333) | - | - |
| Deg. of freedom ($\nu$) | - | 6.2160 (3.4070) | 6.3327 (2.8494) | 7.5412 (4.0468) | - | 11.2065 (4.7433) | 6.8288 (3.8277) |
| Kendall $\tau$ | 0.4320 (0.204) | 0.2990 (0.0267) | 0.2963 (0.0261) | 0.3003 (0.0250) | 0.2255 (0.0236) | 0.1407 (0.0258) | 0.2885 (0.0248) |
| Spearman $\rho$ | 0.6097 (0.251) | 0.4359 (0.0366) | 0.4323 (0.0359) | 0.4378 (0.0343) | 0.3297 (0.0329) | 0.2097 (0.0379) | 0.4215 (0.0343) |
| Tail $\lambda_U$ | - | 0.1418 (0.0531) | 0.1369 (0.0478) | 0.1089 (0.0554) | 0.1334 (0.0452) | 0.0159 (0.0247) | 0.1191 (0.0549) |
| Tail $\lambda_L$ | - | 0.1418 (0.0531) | 0.1369 (0.0478) | 0.1089 (0.0554) | 0.1436 (0.0423) | 0.0159 (0.0247) | 0.1191 (0.0549) |
| | | | | Crisis period | | | |
| Selected copula | t-Student | Frank | Frank | Frank | Gumbel | Frank | Frank |
| Depend. param. ($\theta$) | 0.8087 (0.0369) | 4.1226 (0.6461) | 3.6899 (0.6324) | 4.4063 (0.6063) | 1.5574 (0.1154) | 1.9094 (0.5424) | 3.8021 (0.6472) |
| Deg. of freedom ($\nu$) | 4.1043 (3.5790) | - | - | - | - | - | - |
| Kendall $\tau$ | 0.5996 (0.0390) | 0.3918 (0.0469) | 0.3644 (0.0490) | 0.4174 (0.0425) | 0.3579 (0.0477) | 0.2049 (0.0539) | 0.3731 (0.0500) |
| Spearman $\rho$ | 0.7950 (0.0384) | 0.5619 (0.0605) | 0.5261 (0.0648) | 0.5948 (0.0541) | 0.5089 (0.0623) | 0.3038 (0.0779) | 0.5376 (0.0661) |
| Tail $\lambda_U$ | 0.4948 (0.1022) | - | - | - | 0.4394 (0.0519) | - | - |
| Tail $\lambda_L$ | 0.4948 (0.1022) | - | - | - | - | - | - |

Table 7 displays the results for the copulas selected from tables 5 and 6 with the AIC. Values in brackets represent standard deviations. Besides the copulas' parameters ($\theta$ and $\nu$), the *rank correlation* ($\tau$ and $\rho$) and the asymptotic tail coefficients ($\lambda_U$ and $\lambda_L$) are also presented. Values for these coefficients suggest that, in asymptotic terms, there appears to be more dependence in the pre-crisis than in the crisis period since, by definition, the t-Student copula exhibits tail dependency and the Frank's copula does not. Exceptions are the Canadian and Japanese markets. Whereas the former exhibits asymptotic tail dependence towards the US after the burst of the subprime bubble, the latter reinforced the right hand side asymptotic tail dependence and diminished the left hand side dependence.

Therefore, the results show an increase in global dependence between markets, not always matched by an increase of asymptotic tail dependence, thus suggesting that contagion tests based on the tails of the distributions may produce non-accurate results.

**Results for the assessment of contagion - Test 1:**

Table 8 displays results for the test of existence of financial contagion from the subprime crisis, considering the US market as the focus of the crisis. One thousand replicas were used in the bootstrap procedure (R=1000). In each of the replicas, the obtained values of $\Delta\tau$ (and of $\Delta\rho$) were ordered, leading to a probability function for $\Delta\tau$ (and $\Delta\rho$). This function is then used to calculate the *p-values*, considering as the null hypothesis the non-existence of contagion (i.e. H0: $\Delta\tau \leq 0$). The *p-values* result from a unilateral test, reflecting the left area of probability of $\Delta\tau = 0$.

Table 8. Test of financial contagion with the crisis focus on the US

| Markets | $\Delta\tau$ | p-value | $\Delta\rho$ | p-value |
|---|---|---|---|---|
| USA/Canada | 0.1676*** | 0.0000 | 0.1853*** | 0.0000 |
| USA/France | 0.0928** | 0.0440 | 0.1260** | 0.0410 |
| USA/Germany | 0.0681 | 0.1100 | 0.0938 | 0.1070 |
| USA/Italy | 0.1171** | 0.0140 | 0.1570** | 0.0140 |
| USA/Japan | 0.1324*** | 0.0090 | 0.1792*** | 0.0090 |
| USA/Portugal | 0.0642 | 0.1440 | 0.0941 | 0.1430 |
| USA/UK | 0.0846* | 0.0700 | 0.1161* | 0.0680 |

Note: *, ** and *** mean significance at 10%, 5% and 1% level, respectively.

The test results based on the Kendall's $\tau$ are basically identical to those obtained using the Spearman's $\rho$, thus confirming that the two statistics are substitute. For a 10% significance level, five markets exhibit evidence of contagion: those of Canada, Japan, France, Italy and the UK. The null hypothesis of absence of contagion could not be rejected for the German market (though the values are close to rejection, with a *p-value* of 0,1070 for the test based on the Spearman's $\rho$). The null is clearly not rejected for the Portuguese, thus suggesting that more peripheral markets (perhaps less exposed to the financial products associated to the subprime) are shielded against this crisis contagion effects. In fact, Canada, the closest market to the focus of the crisis, displays the highest level of contagion (*p-value* of 0,0000).

According to what could be anticipated, the markets exhibiting the highest levels of dependence towards the US are also those which are subject to higher contagion intensity. In fact, for the pre-crisis period, the markets exhibiting more synchronized co-movements with the US market are, in decreasing order of the Spearman's $\rho$: Canada (0,6097), Italy (0,4378), France (0,4359), Germany (0,4323), the UK (0,4215), Japan (0,3297) and Portugal (0,2097). This order is almost unchanged if countries are ordered by the *p-values* resulting from the test on the existence of contagion: Canada (0,0000), Japan (0,0090), Italy (0,0140), France (0,0410), the UK (0,0680), Germany (0,1070) and Portugal (0,1430).

Within the European markets presenting similar dependence levels in the pre-crisis period (see Table 7), the German market appears to be the most prepared to resist the crisis, as it presents the weakest signs of contagion (non-significant at the 10% significance level). On the other hand, the Japanese market, in spite of displaying a less intense dependence with the US, vis-avis that of the more developed European countries, appears to be one of the most vulnerable to the crisis effects, as the test to the existence of contagion is significant at the 1% level.

**Results for the assessment of distinct contagion intensity - Test 2:**

In spite of the results of test 1, suggesting that some markets appear to be more affected than others, test 2 is developed to determine if the differences of contagion intensity are statistically significant. Table 9 resumes the test's results.

Table 9. Testing the intensity of financial contagion

| | $\Delta\tau_{A-B}$ | Country B | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Canada | France | Germany | Italy | Japan | Portugal | UK |
| Country A | Canada | | 0.0748 | 0.0995* | 0.0505 | 0.1273 | 0.1034* | 0.0830 |
| | France | | | 0.0247 | -0.0243 | -0.0396 | 0.0286 | 0.0082 |
| | Germany | | | | -0.0490 | -0.0643 | 0.0039 | -0.0165 |
| | Italy | | | | | -0.0153 | 0.0529 | 0.0325 |
| | Japan | | | | | | 0.0682 | 0.0478 |
| | Portugal | | | | | | | -0.0204 |
| | UK | | | | | | | |

Note: *, ** and *** mean significance at 10%, 5% and 1% level, respectively.

The first figure, in the left hand side of the first raw, represents the disparity between the difference of the $\tau$ for the US/Canada pair, between the pre-crisis and the crisis period, and that of the US/France pair: 0,0748 = (0,5996 – 0,4320) – (0,3918 – 0,2990).

In spite of a number of positive figures displayed in the table, suggesting that market A has been more intensely affected than market B[9] (with negative figures suggesting the opposite), the test's results show that the null hypothesis of equal intensity is rejected, and merely at a 10% significance level, only for the pairs Canada/Germany and Canada/Portugal. The Canadian market is the only exhibiting high levels of contagion intensity.[10]

## 5. CONCLUSIONS

This study uses data on MSCI indices to assess financial contagion resulting from the US subprime crisis, adopting the copula theory to characterise and measure dependence links between the US and the other G7 countries plus Portugal.

Two tests are performed to identify the affected markets and the existence of distinct levels of contagion intensity. The first assesses whether evidence of contagion emerges, following of the burst of the subprime mortgage bubble in the US (the ground-zero market), in August 2007. The results suggest that financial markets in Canada, Japan, Italy, France and the UK present significant levels of contagion. The levels for the German market, and mainly for the Portuguese market, are not significant. It is therefore more correct not to refer to contagion in these two cases and simply acknowledge an increase in the dependence towards the US market.

The second test checks if the levels of contagion intensity differ across markets. The results suggest that only in the case of the Canadian market is the level of contagion intensity significantly higher and, even than, only in comparison to the cases of the

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[9] For example, table's first raw suggests that the Canadian market is the most affected, since all the elements in the first raw are positive.

[10] To be precise, one should not refer to increases in the contagion intensity in these concrete cases, given that the markets in Germany and Portugal do not exhibit signs of contagion at a 10% significance level, as may be seen in the results of test 1. It would be more appropriate to state that the contagion intensity in the Canadian market is higher than the increase in the interdependence of the German and Portuguese markets with the US's.

German and the Portuguese markets. Given that, as previously stated, in these two cases there appears to be no significant evidence of contagion, it should be concluded instead that the intensity of contagion displayed by the Canadian market is higher than the increase in the interdependence registered for the German and Portuguese markets with the US, from the pre-crisis, to the crisis period.

Before the development of the two tests, a number of procedures were implemented to increase the robustness of the obtained results. The series of data were first filtered to eliminate effects of temporal dependence, either in the means or in the variances, using ARMA-GARCH models. The standardised residuals of the filtered series, here called filtered returns, were than used to estimate univariate parametric distribution functions, thus ending the process of obtaining the distributions for the marginal variables.

Several copulas were subsequently estimated by the IFM method and the most adequate were selected to represent the co-movements of the pairs of markets under analysis (US-Germany, US-Canada, US-France, US-Italy, US-Japan, US-Portugal and US-UK). The estimated dependence parameters of the various copulas increased from the pre-crisis to the crisis period, thus suggesting that the co-movements between markets have become more pronounced after the burst of the bubble. It was also shown that the levels of dependence of the various markets towards the US are not homogenous, and may be divided in four classes: the first comprises Canada only, the second includes the most developed European markets (Germany, France, Italy and the UK), and the third and fourth contain the Japanese and the Portuguese markets, respectively. In spite of exhibiting distinct levels of correlation, all markets present positive synthetic measures of dependence (i.e. the Kendall's $\tau$), i.e. they appear to be evolving in the same direction.

The t-Student copula was identified as the most adequate to model the dependence structure between markets in the pre-crisis period, whereas Frank's copula appears to be better fitted for the crisis period. Almost all selected copulas present a symmetric structure, in contrast with the results of Longin and Solnik (2001), Ang and Chen (2002), and Ang and Bakaert (2002), who suggest that assets' returns appear to be more correlated in bearish than in bullish markets.

When attention is focused on the levels of asymptotic caudal dependence, the links are stronger in the pre-crisis period, although this is a result of the fact that the t-

Student copula exhibits asymptotic caudal dependence and the Frank's copula does not. Exceptions in this case are the Canadian and Japanese markets: whereas the former exhibits asymptotic caudal dependence with the US market, in the crisis period, the latter reinforced the right hand side asymptotic caudal dependence and diminished the left hand side dependence. Therefore, an increase of global dependence amongst markets is not always accompanied by an increase of asymptotic caudal dependence, thus suggesting that contagion tests that are solely based on the distribution tails may produce non-robust results.

The results show that markets displaying higher levels of dependence in the pre-crisis period also present more robust evidence of contagion, as could be *a priori* anticipated. The Portuguese market displays less signs of contagion, eventually as a result of its more peripheral economic profile. Amongst the European markets presenting similar dependence levels in the pre-crisis period, the German market appears to be the most prepared to resist the effects of the crisis, as it presents the weakest effects of contagion (irrelevant at the 10% significance level). In contrast, the Japanese market, though presenting a less intense dependence towards the US, appears to be one of the most vulnerable, as its evidence of contagion is significant at the 1% level.

The results of this empirical analysis seem to support the operational advantages associated with definition of contagion proposed by Forbes and Rigobon (2001). In fact, the evidence of increased dependence between countries after the crisis should be carefully considered by portfolio managers as it suggests that a simple strategy of geographical diversification may not always be well succeed. Furthermore, the results also support the decisions to inject liquidity on the part of central banks. In theoretical terms, the crisis-contingent theories appear to be the most adequate to explain the transmission of the shock provoked by the US market's crisis.

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